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# A theoretical model of urban roadway pricing, with application to Dallas, Texas

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# A theoretical model of urban roadway pricing, with application to Dallas, Texas

by

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# CHAPTER I: INTRODUCTION

## The Problem

The post-World War II expansion of our nation's transportation network has permitted the domestic economy to progress in a manner inviting an escort of myriad present and future transportation problems. A locational diffusion of employment and residential areas has imposed requirements, increasingly unable to be satisfied by the fixed-line transportation system; as a result, an increasingly greater reliance has been placed upon the automobile to satisfy these requirements. Administrators, in an attempt to alleviate the pressure of an annually augmented demand for urban roadway, have allocated enormous amounts of resources to provide a greater supply of roadway with the result, it has been perceived, of sustaining, as well as enhancing, the problem.

Recognizing the modal interdependence within the transportation sector, and accepting the view that an excessive amount of expansion has been allowed the urban roadway system, new policies have been devised and implemented with the purpose of extracting the commuter from his automobile in favor of more efficient public modes; these policies range from improvements of public modal service qualities to outright bans of automobile parking and use within certain areas. Still other proposals, fashioned typically by economists, have called for the imposition of an urban roadway user price, structured in a way to capture the cost of roadway use, hitherto external to the individual commuter's pocketbook. Such a pricing solution, based upon the marginal cost of roadway use, would, it is maintained, result in a more coordinated relationship among transportation modes.

Within the last ten to fifteen years, a limited group of economists supporting the pricing solution has come to acknowledge the efficiency of utilizing, for the purpose of price estimation, theoretical vehicular traffic movement models, spawned by the last forty years of research by traffic engineers. The present analysis follows this direction, and introduces the use of acceleration noise, a parameter defined by traffic engineers in 1958.

## The Procedure

Observations of the urban problem are elaborated in Chapter II, while the solution offered by the marginal cost pricing principle is reviewed and delineated for application to the present problem in Chapter III. A model is presented in Chapter IV, based upon the fundamental diagram of road traffic and the traffic parameter acceleration noise, both of which are reviewed from the literature of traffic engineering.

Chapter V is addressed to implementation of the model, using two classes of vehicular traffic data provided by the

Texas Transportation Institute, Urban Transportation Systems Program, of Texas A&M University. A comparison of the roadway use toll schedule estimated by the present analysis with previously estimated toll schedules contained in the literature reveals that it is unique in three senses: first, the estimation procedure is based solely upon real-time, rather than historical, data; second, it employs acceleration noise in the estimation of a component of the use toll schedule; and, third, it generates a use toll schedule which is non-monotone with respect to vehicular traffic volume. A number of possible extensions to the present analysis, both economic and engineering, are considered in Chapter VI.

# CHAPTER II: IDENTIFICATION OF THE PROBLEM

## Introduction

The years following World War II have witnessed increased automobile ownership and a concomitant expansion of the nation's system of highways. This transformation of the highway pattern has had a redistributive effect upon households and economic activity within metropolitan areas: decentralization of employment and population, shift of manufacturing and retail establishments to suburban areas, decline in the patronage and service quality of mass transit, increased traffic congestion, and others.

The 1970 census indicates that 85 percent of the nation's population increase during the period 1960-70 was attributable to metropolitan areas (82); associated with the postwar increase of household income has been a preference for residing in the lower density metropolitan locations peripheral to the central city. It is expected that by 1985 these pressures will result in a metropolitan population share of 66 percent claimed by suburban areas (whereas, that share was 50 percent in 1965), while the metropolitan population share is expected to be 70 percent of the nation's population (contrasted with 64 percent in 1965) by that year (37).

Along with this trend toward suburbanization will be a continuing dispersion of urban travel, augmented by an expected doubling of the automobile stock by 1985, and a limited or declining role for public transportation. It has been suggested that by that year there will be a requirement for twice the route miles of urban freeways in existence in 1972 (37).

## Intermodal Competition

Despite a continual expansion of roadway capacity, urban traffic congestion remains a major metropolitan problem. During the period 1950-65, the increase in total vehicle miles in the nation was roughly 4.6 percent compounded annually (57). Associated with this rate of growth, the demand for roadway exhibits a cyclical variation with respect to time. Relative to daily variation on weekdays, peak demands in urban areas generally occur during the hours that most individuals are travelling to and from work (158). The Highway Capacity Manual states that (57, p. 36):

> Because these variations in traffic flow represent patterns of travel desire, the adequacy of a highway cannot be judged by its ability to carry the average volume, but rather must be evaluated in terms of its ability to function properly under peak loads. This concept that the capacity of a highway is a function of both the physical features of the roadway and the pattern of demand shapes present highway practices.

Given this demand for roadway services, the popular solution has been to expand and reconstruct the roadway system.

Any consideration of the transportation sector must recognize that no individual mode is independent of the others (62). In the context of transporting people within an urban setting, the relevant intermodal interdependency is that of the relative service and price characteristics of the private automobile, on the one hand, and the spectrum of public transportation modes, on the other, as observed by the consumer.

Studies of urban travel behavior typically reveal that the consumer deems public transportation a poor substitute for the private automobile regardless of trip purpose (11, 61, 107, 122). For, on the one hand, he does not, generally, estimate in detail the price associated with a particular type of urban trip by automobile; and, when he does estimate it, he equates that price with his variable operating cost plus certain other costs incurred by the trip, such as parking fees and tolls, and disregards the fixed cost of automobile ownership (77, 155). This price illusion, notes Smerk (125, p. 259), "stems from the lack of direct linkage in the mind of the consumer between the actual cost of using his car and the price, as he perceives it, that he pays for a particular trip." Then, on the other hand, he compares this illusory price to the fare charged for that trip by a public mode.

There is considerable support for the view that an excessive amount of resources has been allocated to expansion of the roadway system and that attention should be given to the task of using the existing facilities more efficiently. Under conditions of urban congestion generated by individuals using automobiles during periods of peak demand for urban roadway, a number of devices might be of aid in decongesting the central business district and surrounding urban rings by inducing peak users to abandon their automobiles in favor of public modes, and by diverting some amount of traffic to relatively less congested routes. These devices can be dichotomized into those which increase the appeal of public modes, and those which decrease the appeal of the automobile.

Relative to the former, the alternatives include fare reductions and service quality improvements for public modes. However, the possibility of significantly enhancing patronage via fare reductions appears slight; indeed, a schedule of negative fares might be necessary (25, 78, 95). Studies concerning the impact of improved service quality upon ridership are contradictory, and sometimes indicate a time lag of a year or more before travel habits adjust to quality improvements (18, 25, 74); and, other studies imply, that in order for such renovations to have an appreciable effect, they must be introduced collectively, rather than individually (75, 80).

Relative to the latter variety of devices, the alternatives include the imposition of indirect and direct taxes upon users of the automobile and outright bans of automobile use and parking in certain areas. Indirect taxation, including automobile purchase taxes, annual licenses, fuel taxes and the like, is characterized by the disadvantage that is imposed upon all motorists without discriminating against the use of congested roadway (121). Prohibition of automobile use and parking has the disadvantage of restricting consumer choice (85, 120). Direct taxation -- that is, a congestion tax -- of the individual driver reflects the cost imposed upon vehicles within the stream of traffic and upon the community, as a result of that driver's choice to make a particular trip using his automobile. Considering congestion taxation, Vickrey observes the following (145, pp. 290-291):

> Other proposed methods of pricing to coordinate urban transportation -- among them, parking fees, cordon tolls, special licensing arrangements, and others -- fail to reach the core of the problem. Its solution depends on provision of a direct incentive to the individual driver to economize in the use of high-cost facilities during periods of peak demand and potential congestion. As competition of the private automobile with other forms of urban transportation increases, a rational solution to the pricing of other competing modes depends on adoption of more rational pricing procedures for the private automobile. Without an adequate solution in this area, no fully satisfactory solution in the other areas is possible.

While such direct taxation has been entertained in light of a revenue-producing function, useful in facilitating the objective of roadway investment (17, 91), its concurrent function of rationing roadway has not been sufficiently examined by policy-makers. In the direct taxation approach, however, the revenue-producing function is of subordinate importance and is regarded as merely a beneficial side-effect of the optimization of facility use (13).

One is left, then, with three alternative policy approaches to contend with present and future urban congestion: do nothing, thus allowing congestion to adjust the quantity of roadway supplied and demanded; increase the supply of roadway; and, restrict the quantity of roadway demanded by imposing a price for use of the facility. A choice against the first alternative in favor of the second or third must rest upon the degree of congestion reduction enjoyed, upon the additional benefit derived by those continuing to use the facility at lower levels of congestion, upon the opportunity cost of resources required to expand the roadway, provided that the second alternative is chosen, and upon the "disbenefit to those 'forced off' or affected by those 'forced off' the facilities" should the third alternative be chosen (160, p. 21). Moreover, the choice must rest upon income distribution and equity considerations. The implicit view of the present analysis is that the pricing approach can be appropriate in certain circumstances (90). For, on the one hand, congestion is wasteful of resources (76); provision of a facility designed to satisfy peak traffic demand obviously represents over-investment and might be aesthetically unpleasing in the urban setting; equilibrium is rarely achieved by investment in urban roadway facilities (38, 108). While, on the other hand, information gained from imposition of price could aid city planning (118): urban development projections could be based upon efficient, rather than wasteful, use of facilities (36); zoning and other urban land-use controls could be supplemented by such pricing; and, the loss generated by new facility construction, based perhaps upon faulty prediction of future traffic, could be minimized.

## The Pricing Solution

Increasingly, economists agree that before more efficient use of urban roadway can be achieved within, as well as among, the various modes of urban travel as a whole, specific attention need be given the private automobile, which accounts for an ever-expanding share of that facility's use.

The price to be applied to the private automobile should reflect the marginal cost of roadway use. Under existing conditions, the individual user of urban roadway, faced with alternative ways to achieve the objective of his trip, takes into account only the cost which each alternative imposes upon him, but is not aware of the incremental cost imposed upon others by each alternative.

The customary diagram employed to represent the market for roadway services is presented as Figure 1 (4, 73, 139, 149, 160, 162). The curve AC represents the average cost per vechicle using some portion of roadway; it is the total cost of all vehicle units using the roadway divided by the total number of those vehicle units. It slopes upward as traffic flow (i.e., the number of vehicle units passing a point per unit time) increases, reflecting increases in unit cost due to increases in vehicle interaction. The marginal cost, depicted

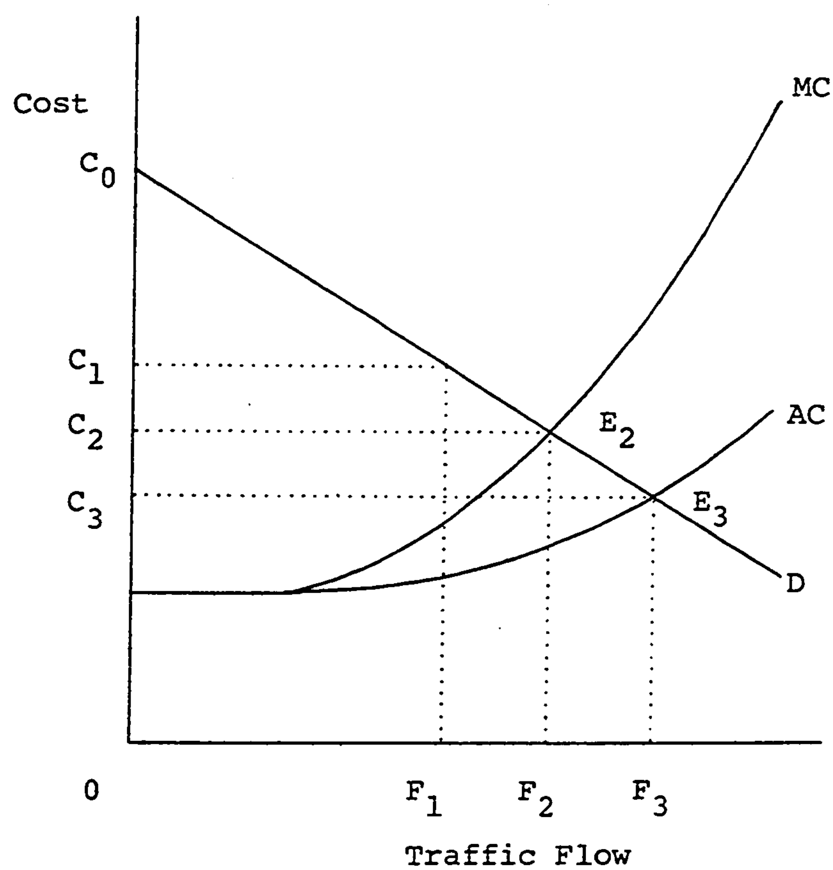


Figure 1. Traditional graphical representation of the roadway market

by MC, represents the increase in total cost due to an increase in traffic flow, and is composed of two elements: namely, the cost borne by the additional vehicle (that is, the average cost) and the cost which that vehicle imposes upon the other vehicles (that is, the vertical distance between AC and MC). The latter element of marginal cost is external to the additional vehicle, but is internal to the total vehicle units using the section of roadway; that is, it is the social cost levied by the additional vehicle upon the other vehicles.

Drivers respond to higher roadway-use cost generated by greater traffic flow on the roadway section by using the section to a lesser extent; this responsiveness is reflected by the downward-sloping demand curve, represented by D. This curve specifies the amount an individual vehicle is willing to pay for use of the section at any given level of traffic flow.

Given that circumstances warrant the imposition of a pricing mechanism to optimize use of the roadway section, the vertical distance between AC and MC delineates the price to be levied at various levels of flow. This price would internalize to the additional vehicle the social cost imposed by it upon the others, and typically is not recognized by conventional pricing schemes. Consideration of this social cost must include both traffic flow differences among spatially separated sections of roadway, as well as temporal differences on one particular section.

Without imposition of a price for use of the facility, equilibrium will be reached at point $E_3$. Interpreting D as the marginal benefit curve and MC as the marginal social cost curve, it follows that whenever traffic flow is greater than $OF_2$, imposition of a price to effect an approach toward equilibrium at point $E_2$ increases net social benefit, the difference between the area under the D curve, and that under the MC curve; economic efficiency is improved by shifting the equilibrium from point $E_3$ to $E_2$. Likewise, when traffic flow is less than $OF_2$, say $OF_1$, efficiency is improved by encouraging flow to increase, approaching equilibrium at point $E_2$. Any divergence of flow from $F_2$ results in losses to the economy as a whole.

This diagrammatic approach traditionally stresses the following four points. First, it is addressed to the allocation of roadway service, taken as a scarce resource. Second, it assumes that prices and individuals' incomes are constant.

Third, it is directed solely to the private vehicle. Other roadway users, namely the pedestrian and the user of public modes, are ignored. Similarly ignored is the entire spectrum of community social costs, that is, the costs generated by increases of traffic flow, external to the total vehicle units using the section of roadway, but internal to the community (80). Winch classifies these costs into four groups, relative to immediate incidence (156, pp. 15-16):

> (1) the costs of the highway itself including construction, maintenance, and operation, which fall initially on the highway authority, together with those opportunity costs such as property taxation which would rest on the highway authority were they levied; (2) the costs of vehicle operation which fall on those responsible for the operation of the vehicles; (3) the users' personal costs of time, inconvenience, and risk, which fall on the persons travelling or the owners of goods travelling; and (4) the costs which fall on the community at large or sections of it.

That is, the traditional approach is concerned with the types of costs subsumed within groups (2) and (3).

Fourth, "the analysis proceeds in terms of <u>expected</u> average demand and cost relationships over a period of time.... The analysis does not purport to indicate an optimum position in respect of each particular occasion in time when flows increase. Hence means for restraint indicated by these arguments will be the more useful the more regular traffic patterns, desires for trips, etc., are" (7, p. 190). While the approach stresses that equilibrium should be maintained at the intersection of the demand and marginal cost curves, it fails to suggest how the optimal price is to be determined in practice.

Optimization of the urban roadway market, it has been seen, can be secured by imposition of a price structure based upon congestion costs. Given the periodic fluctuation of urban traffic, such a price would not be computed from cost-of-roadway considerations, but would be homologous to the fluctuation of the congestion costs generated by the time pattern of traffic. "In such circumstances, congestion costs become the measure 'par excellence' of what a price structure arrived at by the normal processes of the market would tend to be" (142, p. 109).

# CHAPTER III: MARGINAL COST PRICING

## Introduction

A set of prices might be employed to simultaneously accommodate three issues: the rationing problem, the investment problem, and the income distribution problem. When pricing urban roadway, the investment problem is relegated an ancillary position; moreover, the present analysis abstracts from the solution of the distribution of income.

An increasing proportion of trips made on urban and, particularly, central business district roadway consists of work-trips; and, due in part to the institutionalized nature of employment, work-trips characteristically strain facility capacity during periods of peak demand, while trips generated for noncommuting purposes (that is, shopping, business, and recreation) are made during off-peak hours. Relative to the choice of mode, the basic determinants are speed, convenience, comfort and price. In all but the largest urban areas, the private automobile usually dominates noncommuter trips to the central business district (90, p. 88):

> Commuting, on the other hand, is likely to be a much more complex matter. Where the costs of private automobile operation are not too high, either in money or in traffic congestion, commuters too may prefer the automobile; elsewhere they will presumably find public transportation more attractive. Also, rising incomes presumably will work to the advantage of auto commuting.

Therefore, this analysis will solely consider the home-to-work and work-to-home trips, which use the mode of private automobile.

## The Application of Marginal Cost Pricing

The historical development of the marginal cost pricing principle is well-documented within the literature of economics (21, 86, 111, 112, 114, 140, 148). While some economists take the position that the principle is not workable in application (22, 159), others feel that it must play a principal role in any pricing design which has the goal of efficient utilization of resources (15, 79, 147).

With particular application to the problem of establishing a set of prices for roadway use, the Smeed Report considers the following to be important operational requirements of the pricing scheme selected (123, p. 7):

(1) Charges should be closely related to the amount of use made of the roads.

(2) It should be possible to vary prices to some extent for different roads (or areas), at different times of day, week or year, and for different classes of vehicle.

(3) Prices should be stable and readily ascertainable by road users before they embark upon a journey.

(4) Payment in advance should be possible, although credit facilities may also be permissible under certain conditions.

(5) The incidence of the system upon individual road users should be accepted as fair.

(6) The method should be simple for road users to understand.

(7) Any equipment used should possess a high degree of reliability.

(8) It should be reasonably free from the possibility of fraud and evasion, both deliberate and unintentional.

(9) It should be capable of being applied, if necessary, to the whole country....

The literature related to the topic generally concurs with these requisites (119, 142).

It has already been noted that automobile travel cost can be categorized into four types: the costs of the highway itself, the explicit costs borne by the automobile operator, the personal costs borne by the automobile operator, and the community costs. The costs of the highway itself include development and construction, fixed maintenance, administration, and interest on capital. The explicit costs are just the costs of automobile ownership and operation, while the personal costs include the operator's travel time, accident risk, and general inconvenience. Community costs include air and noise pollution and other loss of amenity, as well as that portion of roadway maintenance cost attributable to traffic.

Now, while the first type of travel cost is purely a long-run cost, the last three are short-run, as well as long-run in nature. The only costs relevant to the price-output decision-making process in the context of urban roadway, are those costs that vary with traffic flow -- short-run costs (65, 144). This analysis is addressed to the problem of estimating the short-run explicit and personal costs of automobile travel, for purposes of drafting a price for urban roadway use.

As demonstrated in Figure 1, the existence of external costs has the following consequence (51, p. 16): "a larger than optimum number of trips are taken because the number of trips will be determined by the intersection of the demand function with the average private cost function rather than with the marginal social cost function." To the extent that the short-run explicit and personal costs of automobile travel (that is, again, automobile operating cost, travel time, and accident risk) are generated by traffic congestion, they are external to the individual vehicle, yet internal to the total vehicle units; and, since this analysis ignores the short-run community social costs (that is, again, air and noise pollution, loss of amenity, and so on) -- that class of costs external to the total vehicle units using the section of roadway under consideration, yet internal to the community -- the average cost and marginal cost curves of Figure 1 become, respectively, the short-run average cost and short-run marginal cost curves, represented by SRAC and SRMC, in Figure 2. That

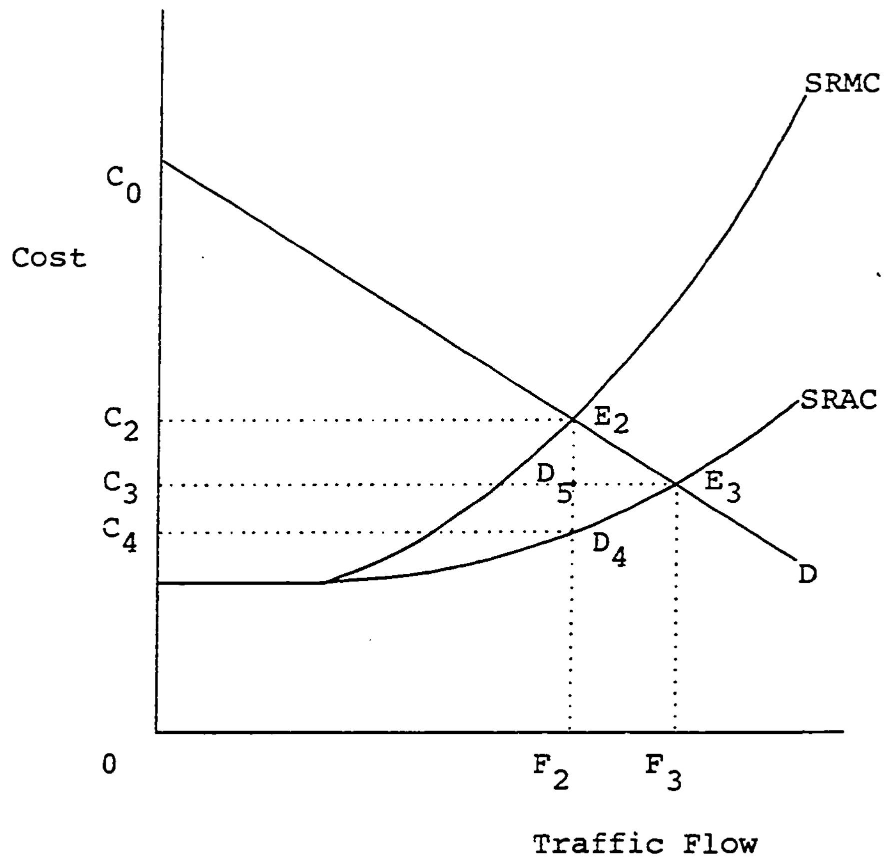


Figure 2. The roadway market and net benefit

is by definition, the SRAC and SRMC curves are the short-run average and marginal social costs; and, the SRAC is also the short-run average private cost curve, since it ignores those costs which are external to the total vehicle units; moreover, it is also the short-run marginal private cost curve, since it portrays the marginal explicit and personal costs sustained by additional vehicles on the roadway section.

The consideration of Figure 1 noted that any divergence of vehicular system flow from $F_2$ results in a loss of net benefit to the economy as a whole. In Figure 2 with the imposition of a roadway user price, delineated by the difference between SRMC and SRAC, the SRMC curve becomes the relevant decision-making function of the individual vehicle; equilibrium occurs at intersection point $E_2$, with traffic flow $OF_2$, and with net benefit equal to area $C_4C_0E_2D_4$ (that is, total benefit $OC_0E_2F_2$ minus total cost $OC_4D_4F_2$). Whereas in the absence of such a price, the SRAC curve is relevant, equilibrium occurs at intersection point $E_3$ with traffic flow $OF_3$, and with net benefit equal to area $C_3C_0E_3$ (that is, total benefit $OC_0E_3F_3$ minus total cost $OC_3E_3F_3$). The roadway user price imposition, therefore, increases net benefit, since the reduction in total benefit (that is, $OC_0E_3F_3$ minus $OC_0E_2F_2$ equals $F_2E_2E_3F_3$) is diagrammatically less than the reduction in total cost (that is, $OC_3E_3F_3$ minus $OC_4D_4F_2$ equals $C_4C_3E_3F_3F_2D_4$); that is, the deduction of $E_2E_3D_5$ (the net benefit enjoyed by the marginal vehicle units occupying the roadway when flow is $OF_3$) is less than the addition of $C_3C_4D_4D_5$ (the net benefit enjoyed by all vehicle units occupying the roadway when flow is $OF_2$). At equilibrium $E_2$, each vehicle is assessed the user price $C_4C_2$.

Let it be noted, however, that this solution ignores this user price in the sense that it implicitly assumes it to be refunded to the user. In the absence of this unrealistic assumption -- unrealistic since prior knowledge of such a future refund on the part of the prospective user precludes the disincentive provided by the user price in the absence of such a refund -- the facility users suffer a loss of net benefit. For with the imposition of the user price, equilibrium occurs at $E_2$, and net benefit is represented by the area $C_2C_0E_2$ (that is, total benefit $OC_0E_2F_2$ minus total cost $OC_2E_2F_2$); while in the absence of the price, net benefit is represented by $C_3C_0E_3$ (that is, $OC_0E_3F_3$ minus $OC_3E_3F_3$): the imposition of the user price decreases net benefit of facility users by the amount represented by area $C_3C_2E_2E_3$ (that is, $C_3C_0E_3$ minus $C_2C_0E_2$).

Under the imposition of user price $C_4C_2$, all of the facility users under equilibrium $E_2$ and flow $OF_2$ suffer economic loss (160, p. 22) as compared to the situation under equilibrium $E_3$ and flow $F_3$. For under the marginal cost pricing policy at $E_2$, they sustain the total disbursement $OC_2$ (that is, unit cost $OC_4$ plus user cost $C_4C_2$); whereas under the average cost pricing policy at $E_3$, they sustain only the user cost $OC_3$: the imposition of the user price decreases their unit cost by the amount $C_4C_3$ (that is, $OC_3$ minus $OC_4$), but increases their total disbursement by $C_4C_2$ (an amount exceeding the reduction of unit cost by $C_3C_2$). However, whereas the users -- individually and in aggregate -- suffer an economic loss, the economy as a whole enjoys an increase in net benefit, since it extracts from facility users the amount of net benefit represented by area $C_3C_2E_2E_3$. Thus, net benefit to the economy as a whole is maximized at equilibrium point $E_2$.

To reiterate, this analysis ignores the short-run community social costs. The inclusion of these costs could be expected to shift the SRAC and SRMC curves upward, resulting in a higher user price and lower traffic flow in equilibrium.

## Peak-load Pricing

The demand for the services of urban roadway, as well as of any public utility, "varies not only periodically over the month and year, but also varies significantly on a daily or weekly basis.... The problem of meeting these variations in load with some optimum sized plant capacity and the accompanying investments and costs, all in the framework of a pricing structure, is called the peak-load pricing problem" (106, p. 304). The problem classically arises in the context of a market, the commodity of which is not storable from a technological point of view; electric power and transportation are notable examples. Since the economics of the problem relates to the design of the pricing policy which results in the efficient utilization of the correct capacity, while covering the social cost of resources supporting the facility, the use of marginal cost pricing is apparent (32, 92, 105, 129).

With respect to the transportation sector, Vickrey observes that (146, p. 452):

> ...in no other major area are pricing practices so irrational, so out of date, and so conducive to waste as in urban transportation. Two aspects are particularly deficient: the absence of adequate peak-off differentials and the gross underpricing of some modes relative to others.... In nearly all other operations characterized by peak load problems, at least some attempt is made to differentiate between the rates charged for peak and for off-peak service.... But in transportation, such differentiation as exists is usually perverse.

Conventional peak-load pricing calls for a known periodic price schedule if the service is storable; but in the case of a nonstorable service the problem is more interesting, in that short-run marginal cost pricing calls for stochastic price changes as a function of stochastic demand. Notwithstanding the administrative and legal arguments against stochastic price variation in the public utility field, economic efficiency favors a pricing policy based not only upon the realization that the quantity demanded of the service is a function of its price, but also upon the predisposition of frequent price change.

## The Roles of Price

Characteristically, civil administrators and engineers maintain that the roadway use price is a potentially weak tool of traffic flow control and offers little help in the solution of the urban transportation problem (24, 127). On the contrary, economists acknowledge the use of such a price in three related roles: the rationing of existing facilities, investment in new facilities, and the distribution of income.

Given the goal of rationing the existing roadway facilities so as to maximize net benefit in the sense described in Figure 2, a schedule of user prices based upon marginal cost is required to maximize the benefit of the facility to peak-hour users by encouraging its use by those who experience a relatively high level of explicit and personal costs while travelling the roadway, and by discouraging its use by those who experience a relatively low level of those costs (6, 26, 34, 36, 124, 134, 137). Such a set of user prices promotes efficient use of existing facilities by permitting the consumer of roadway to determine the mode for the trip and to determine, if use of his automobile is indicated, the route of the trip. That is, it enhances the coordination of modes and routes (145).

If the marginal cost principle provides the basis of a schedule of user prices, those prices will fail to solve the investment problem to the extent that the section of roadway subject to user pricing exhibits increasing returns to scale (as in the case of rural roadway); for, the solution of the rationing problem will not attract total revenue sufficient to fund improvements to the facility. Similarly, the solution of the rationing problem will generate a surplus of total revenue to the extent that the facility exhibits decreasing returns to scale (as in the case of urban roadway) (3, 45, 102, 110, 163).

The marginal cost pricing approach, however, discards any relationship between revenue derived and investment; government is called upon to subsidize any deficit and dispose of any surplus, so as not to interfere with the rationing role of the user price. Yet, although the decision-making process related to the investment role must be based upon analyses divorced from the rationing role, the feedback generated by the latter is crucial in the former, aside from the actual existence of revenue generated: the pattern of prices levied should be employed as data of benefit-cost analyses of facility expansion. The prices thereby promote efficient solution of the investment problem (96, 132, 143).

Likewise, the marginal cost pricing approach treats roadway efficiency and income distribution as independent problems (40). If a set of user prices solves the rationing problem, a certain distribution of wealth is dictated; and, unless one is content to accept any particular distribution resulting, these two roles are in apparent contradiction. The approach can circumvent the problem in a number of ways (132, p. 115):

> One way is to suppose that any distribution of income is ethically as good as any other, so that the problem is essentially ignored. Another solution, differing only subtly from the first, is to suppose that there are both good and bad distributions of income but that whatever distribution is generated by a price system that solves the...rationing problem, is the best one. A third device for avoiding the problem is to assume that the distribution of income can be altered without tampering with the price mechanism. This is conceptually possible.

Relative to the third device, Vickrey (142, p. 117) feels that there "would seem to be no overwhelming difficulty, indeed, in coupling the institution of congestion charges with adjustments in income tax rates which would leave every income group as a class better off." And further, Winch (156, p. 38) maintains that there "is certainly no reason for using the planning and financing of one particular economic activity, such as highway construction, to redress any remaining inequities in the distribution of income, especially when there is no way of knowing whether, and if so where, such inequities exist."

The approach of this analysis, then, is to acknowledge the rationing role of roadway user prices, while abstracting from the investment and distribution roles.

# CHAPTER IV: THE COSTS OF CONGESTION

## Introduction

Traditional economic theory supports the fact that in the short run the individual driver incurs an increase in congestion cost as the flow of traffic increases. However, the theory fails to take account of the individual components of that cost, it fails to specify the relationship of those components with traffic flow, and it fails to resolve the problem of how an economically optimal congestion toll is to be determined in practice.

The approach of the present analysis is to synthesize previously developed pricing principles of economic theory with certain deterministic relationships of traffic flow theory, the latter being employed to investigate the components of congestion cost. Acceleration noise, a traffic parameter relatively recently developed by traffic theorists, is introduced and utilized for the purpose of enhancing the administrative feasibility of urban roadway pricing.

## The Theory of Traffic Flow

### The fundamental diagram of road traffic

The "theory of traffic flow" generally refers to that body of knowledge concerned with the theoretical analysis of vehicle movement over roadway. When examining the properties of the traffic stream, one needs to distinguish between two ways of viewing that stream: the macroscopic and the microscopic approach. In the case of the former, the properties are specified from a global point of view by viewing the traffic stream as one unit; while in the case of the latter, those properties are specified from a local point of view, based upon the action of one vehicle unit operating within the traffic stream.

Three deterministic traffic variables are defined, as follows:

1. flow -- the number of vehicle units passing a point of roadway during a unit of time;
2. concentration -- the number of vehicle units occupying a unit length of roadway at a point in time; and,
3. space mean speed -- the average speed of the number of vehicle units occupying a unit length of roadway at a point in time, where speed is the distance travelled by a vehicle unit during a unit of time.

   Vehicle units are assumed to be a uniform length.

Equation 1 is the generic relationship among flow, concentration, and space mean speed:

$$q = ku \qquad (1)$$

where:

$q$ = flow;

$k$ = concentration;

u = space mean speed.

The relationship has the general form illustrated in Figure 3, which has been termed by Haight (46) the "fundamental diagram of road traffic". Since, by Equation 1, space mean speed (henceforth, u will be referred to simply as "speed") is the ratio of flow to concentration, the slope of vector $u_m$ represents the speed associated with flow $q_m$ and concentration $k_m$; similarly, any speed within the range of the diagram is defined to be the ratio of the associated flow and concentration. Among the three variables, only concentration has a theoretical maximum; this maximum, "jam concentration", is denoted by $k_j$, and corresponds to solidly packed vehicle units. While the other two variables have observed maximums, absolute maximums are not assigned. An examination of the boundary conditions of the fundamental diagram reveals these observed maximums: relative to speed, $u = u_f$ at $k = 0$, and $u = 0$ at $k = k_j$; the speed $u_f$ denotes the "mean free speed" of the system, that is, the speed approached by vehicle units as flow and concentration approach zero, and interaction among vehicle units diminishes; relative to flow, $q = 0$ at $k = 0$, and $q = 0$ at $k = k_j$. Hence, as concentration increases from $k = 0$, speed monotonically decreases from the observed maximum, $u_f$; flow increases until it attains the observed maximum, $q_m$, at $k = k_m$, and goes to zero as concentration increases from $k_m$ to $k_j$.

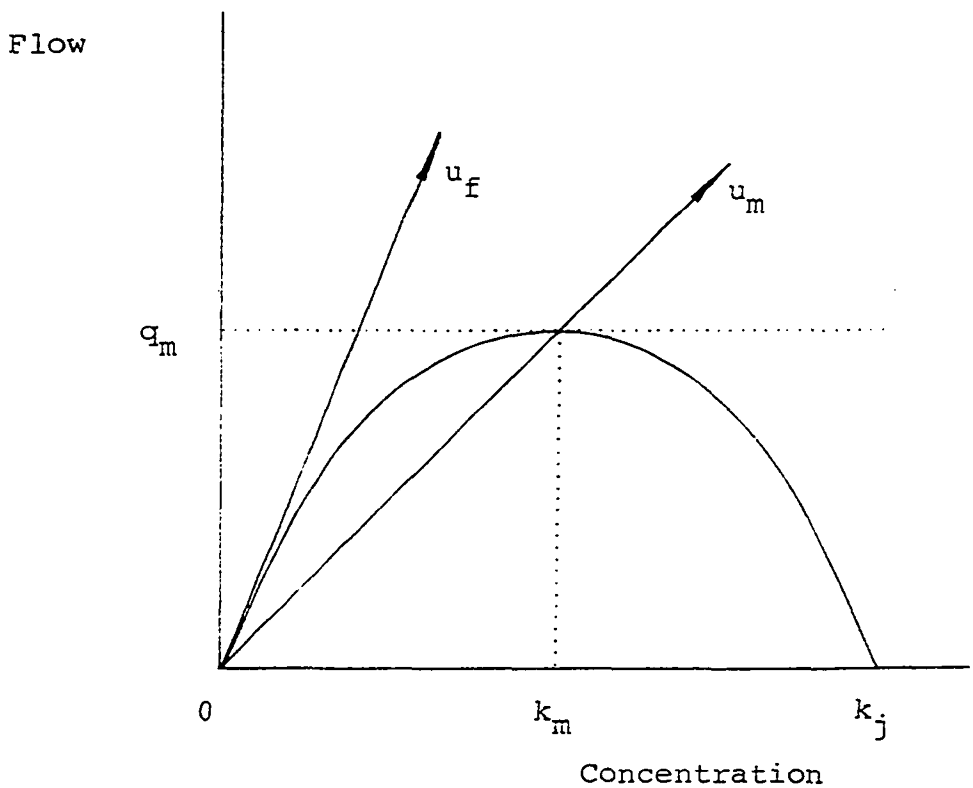


Figure 3. The fundamental diagram of road traffic

Within the limits set by these boundary conditions lies the fundamental diagram. "Nevertheless, it would be a mistake to suppose that any one particular...diagram will apply in all situations. It is characteristic of a particular place at a particular time with a particular population of drivers. Any one of a number of events could change its form: changing visibility at twilight, a sudden fall of rain, or even the appearance of a police car" (46, p. 72).

Various approaches have been used to deduce the functional form of the fundamental diagram; a brief survey follows. Greenshields (44) in 1934 -- one of the earliest pieces in the literature -- noticed a linear relationship between concentration and speed, based upon empirical investigation of the macroscopic properties of traffic. This condition of linearity can be stated as

$$\frac{u}{u_f} + \frac{k}{k_j} = 1. \qquad 2$$

That is, a speed of x% of free speed is associated with a concentration of (1-x)% of jam concentration. And hence, the linearity condition implies the following:

from Equation 2,

$$u = u_f(1 - \frac{k}{k_j}) ,$$

or

$$u = u_f - (\frac{u_f}{k_j})k; \qquad 3$$

substituting Equation 1 into this result,

$$\frac{q}{k} = u_f - (\frac{u_f}{k_j})k,$$

or

$$q = u_f k - (\frac{u_f}{k_j})k^2 \ . \qquad 4$$

Thus, if a linear relationship between concentration and speed is assumed, then the relationship between flow and concentration is parabolic.

To determine the speed and concentration, $u_m$ and $k_m$, respectively, at which flow is at its observed maximum, $q_m$, Equation 2 is substituted into Equation 1 to yield the following:

$$q = ku$$

$$= \frac{k_j}{k_j} ku \frac{u_f}{u_f}$$

$$= k_j u_f (\frac{u}{u_f}) (\frac{k}{k_j})$$

$$= k_j u_f (\frac{u}{u_f}) (1 - \frac{u}{u_f}) \ .$$

Taking the partial derivative of this result with respect to speed

$$\frac{\partial q}{\partial u} = \frac{1}{u_f}(k_j u_f - 2k_j u)$$

and setting it equal to zero

$$\frac{1}{u_f}(k_j u_f - 2k_j u_m) = 0$$

yields

$$u_m = \frac{1}{2} u_f. \qquad 5$$

Similarly, Equations 1 and 2 imply that

$$q = ku$$

$$= k_j u_f (1 - \frac{k}{k_j}) (\frac{k}{k_j});$$

taking the partial derivative with respect to concentration and setting it equal to zero

$$\frac{1}{k_j}(u_f k_j - 2u_f k_m) = 0$$

produces

$$k_m = \frac{1}{2} k_j. \qquad 6$$

That is, the speed and concentration associated with the observed maximum flow are one-half the observed maximum speed and one-half the absolute maximum concentration, respectively. Substituting Equations 5 and 6 into Equation 1 produces

$$q_m = \frac{1}{4} k_j u_f. \qquad 7$$

Following the direction provided by Greenshields, empirical investigation of the macroscopic properties of traffic continued under the linearity assumption (35, 50, 66, 83, 84, 97, 101) despite the lack of a firm theoretical basis. Other researchers, prompted by the instability of the linear concentration-speed relationship at certain concentration levels, entertained the possibility of two or more distinct linear regimes (28). Then in 1959, Greenberg repudiated the linear assumption in favor of a natural exponential function (41, 42, 54); his deduction was based upon a theoretical macroscopic investigation, wherein vehicular traffic flow was considered analogous to a one-dimensional continuous fluid flow. Independent inquiry based upon microscopic theorization (161) led to the same form for the fundamental diagram.

All of these empirical and theoretical attempts to deduce a functional form of the fundamental diagram, some at the macroscopic level and some at the microscopic, are of a deterministic nature; solely for the purpose of a complete -- albeit terse -- survey, it must be mentioned that stochastic models are equally suitable deductive tools (46).

There have been suggested for the fundamental diagram, therefore, a host of functional forms other than those introduced by Greenshields and Greenberg -- one could compile a sizeable list of references related solely to this question of the "correct" form. In order to resolve the question, one simultaneously considers theoretical as well as empirical aspects.

Relative to the theoretical aspects, four boundary value postulates of the fundamental diagram have already been examined: the value of u must be $u_f$ at $k = 0$, and must be zero at $k = k_j$; and, the value of q must be zero at both $k = 0$ and $k = k_j$. A fifth postulate might be examined. For, the first postulate -- namely, $u = u_f$ at $k = 0$ -- is able to be better stated as

$$\lim_{k \to 0} u = u_f;$$

however, the "existence of such a limit is more a matter of common sense than of mathematical demonstration or even empirical verification" (46, p. 87). One might extend this supposition and assume that

$$\lim_{k \to 0} \frac{du}{dk} = 0;$$

this condition is, then, a fifth postulate of the fundamental diagram. Considering, now, the form of the Greenshields model given as Equation 3, the first four postulates are straightforwardly verified. However, the fifth is not satisfied:

$$\lim_{k \to 0} \frac{du}{dk} = \lim_{k \to 0} \left(- \frac{u_f}{k_j}\right) \neq 0.$$

If one is willing, though, to replace postulate five with the weaker form

$$\lim_{k \to 0} \frac{du}{dk} = \delta,$$

where $\delta$ is "sufficiently" small, then the Greenshields model is theoretically satisfying. It should be impressed that relatively few functional forms suggested for the fundamental diagram fare as well!

Relative to the empirical aspects, the spirit of empirical research requires one to employ the standard tests of statistical significance when choosing among the host of functional forms; and, moreover, one is not interested in specifying "the true functional form" of the fundamental diagram for the particular data under analysis, but rather, in specifying "a good predictive functional form". Most of the engineering studies in the literature are "of the latter type and have suggested linear relationships that are sufficient for the purposes of the analysis" (161, p. 338).

## Level of service

Economists and engineers have long been concerned with the problem of measuring the value of roadway. The Highway Research Board, Committee on Highway Capacity suggests, that research related to the development of such a measure be directed toward the idea of roadway "level of service" (57, p. 7):

> 'Level of service' is a term which, broadly interpreted, denotes any one of an infinite number of differing combinations of operating conditions that may occur on a given lane or roadway when it is accommodating various traffic volumes. Level of service is a qualitative measure of the effect of a number of factors, which include speed and travel time, traffic interruptions, freedom to maneuver, safety, driving comfort and convenience, and operating costs.

In an early paper on the topic of roadway level of service, Carmichael and Haley (16) note an inverse relationship between traffic flow and fuel economy, and a direct relationship between braking (expressed in terms of seconds per mile) and traffic flow in urban street operation. Hall and George (48, p. 511) examine the effectiveness of travel time as a measure of congestion and quality of urban traffic service "not only to the day-to-day operations of the streets, highways, and freeways, but also to the long-range determination of a practical and attainable level of service"; they express the quality of service in terms of average over-all speed, and relate it to street geometrics and functional classification.

Greenshields proposes the following expression for traffic flow quality:

$$Q = \frac{S}{s\sqrt{f}}$$

where:

Q = quality index;

S = average speed (miles per hour);

s = absolute sum of speed changes per mile;

f = number of speed changes per mile.

This approach is based upon the contention that the overall speed of a vehicle unit operating upon a section of roadway determines travel time and is, therefore, directly related to flow quality; while, the amount and frequency of speed changes generate driver irritation, increase cost of vehicle operation, and are, therefore, inversely related to flow quality. Greenshields notes (43, p. 5) that the "chief characteristic of the quality index number is that it is basically equal to speed divided by change of speed...."

Platt (104), in deference to the Committee on Highway Capacity recommendation related to level of service, incorporates a version of the Greenshields quality index within a relatively cumbersome index. The index is proposed on a purely theoretical foundation and is designated as a function of the quality of traffic flow, driver satisfaction, driver effort, and driver annoyance due to delay; the variables suggested to measure these terms include the vehicle unit's average speed, speed change rate, change of direction rate, accelerator change rate, brake application rate, total trip time, and running time.

Concomitant with this general direction is the approach introduced by Herman et al. (19, 55). Taking the position that speed dispersion among vehicle units is related to the mean of some measure of traffic flow resistance averaged over all drivers, those authors speculate that (19, p. 183) "a quantity sensitive to the resistance to flow is the acceleration noise experienced by a given vehicle. We define this noise as the dispersion in the acceleration distribution function." Interest in acceleration noise has grown since empirical research (67, 72, 136) has verified that the concept is related to the three primary elements of the traffic stream -- the operator of the vehicle, the roadway geometrics, and the traffic condition -- and that it is a measure of smoothness of flow of the traffic stream; interest has also grown since theoretical research has demonstrated a relationship between acceleration noise and the fundamental diagram of road traffic.

When an individual operating a motor vehicle on a relatively straight and level section of urban roadway encounters a condition of traffic volume light enough to allow him to maintain his desired speed, his accelerations and decelerations are due solely to his inattentiveness, and his distribution of acceleration about a mean acceleration, $a_{ave}$, would resemble Figure 4(a), in which the variation about the mean is relatively small. Everything else equal, if traffic volume increases to a level which increases the number of vehicle units the individual encounters to the degree that in order to maintain his desired speed he is forced to engage in more frequent and more violent maneuvers -- namely, changing lanes, passing other vehicles, weaving, and the like -- his distribution of acceleration about a mean acceleration would resemble Figure 4(b), in which the variation about the mean is relatively high. The latter case is characterized by a higher level of acceleration noise than is the former. It will be seen that the square of acceleration noise is analogous to the second (sample) moment,

$$m_2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \overline{X})^2 ,$$

where n denotes the number of observations in the sample, $X_i$ denotes the $i^{th}$ observation, and $\overline{X}$ is defined to be the arithmetic mean of the sample values.

Jones and Potts (72) have mathematically developed the definition of acceleration noise, using the following approach. If $u(t_i)$ and $a(t_i)$ denote the speed and acceleration, respectively, of a vehicle unit at time $t_i$, then the average acceleration of the vehicle unit during a trip of duration T is

$$a_{ave} = \frac{1}{T} \int_0^T a(t_i)dt$$

$$= \frac{1}{T}[u(T)-u(0)] ,$$

(a)

(b)

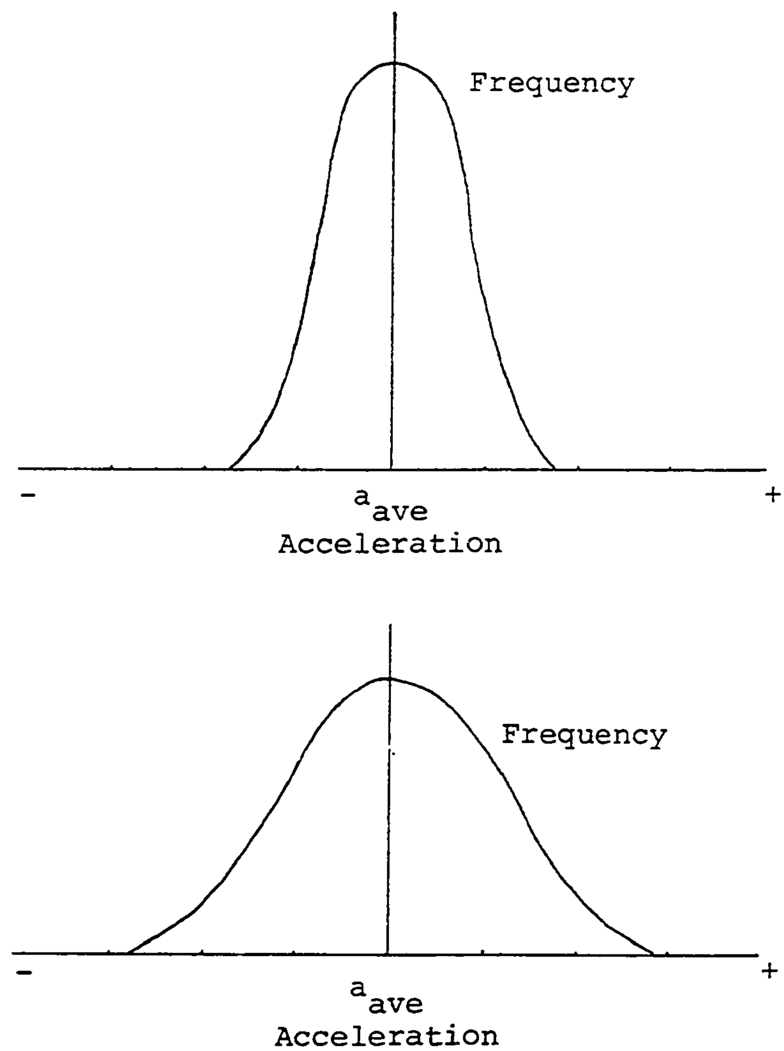


Figure 4. Frequency distribution of acceleration under conditions of (a) light and (b) heavy traffic volume

where $u(0)$ = the initial speed;

$u(T)$ = the final speed.

The integral over a time T of the squared differences between $a(t_i)$ and $a_{ave}$ is

$$\int_0^T [a(t_i)-a_{ave}]^2 dt;$$

and, the acceleration noise is defined to be

$$\sigma = \{\frac{1}{T}\int_0^T [a(t_i)-a_{ave}]^2 dt\}^{1/2} . \qquad 8$$

## The Basic Model

### Introduction

Traffic flow theorists have come to rely heavily upon an analogy between vehicular traffic flow and fluid flow, for this analogy allows one to describe the properties of vehicular traffic by means of certain principles of fluid mechanics, and to demonstrate the relationship between acceleration noise and the fundamental diagram of road traffic. This relationship, having been reviewed (14, 29, 30, 31), will be applied to the problem of urban roadway pricing.

### The fluid analogy -- the fundamental diagram

The fundamental theory of fluid mechanics states that all fluid situations satisfy the principle of continuity: that is, "the fact that net mass inflow into any small volume per unit time must just equal its time rate of increase of mass" (131, p. 80); the principle is based upon "two fundamental propositions:

(a) The mass of the fluid is conserved, i.e. the fluid is neither created nor destroyed in the field of flow.

(b) The flow is continuous, i.e. empty spaces do not occur between particles which were in contact.

The status of these propositions is different, for the first is always true, whereas the second is an assumption about the nature of the flow which, in fact, is sometimes violated" (33, p. 41).

The analogy between the flow of fluid and the movement of vehicular traffic is based upon the assumption that vehicular traffic flow behaves as a continuous fluid flow. Two considerations preclude the application of the analogy to all traffic conditions: the principle of conservation of mass -- proposition (b) -- suggests that the analogy holds only for highly-concentrated traffic; and, the fact that individual vehicle units are individually controlled is not taken into account. Yet, traffic flow theorists feel that these considderations need not be of major concern since "most important traffic-control problems occur only under high-density and other than 'free movement' conditions..." (161).

One may conceive of the principle of continuity by imagining a two-dimensional space through which there occurs a one-dimensional flow; this imaginary control system is fixed in position and offers no resistance to the flow. The application of the principle of conservation of mass to this system implies that the net flow from the space equals the change in mass within the space; that is (153, pp. 8 and 96), the principle can be stated as follows:

$$\left\{\begin{array}{l}\text{the time rate of change}\\ \text{of mass of the space}\end{array}\right\} = \left\{\begin{array}{l}\text{the rate at which mass is}\\ \text{supplied to the space}\end{array}\right\}$$

or, as

$$\left\{\begin{array}{l}\text{the time rate of change}\\ \text{of mass of the space}\end{array}\right\} = \left\{\begin{array}{c}\text{mass flux}\\ \text{into the}\\ \text{system}\end{array}\right\} - \left\{\begin{array}{c}\text{mass flux}\\ \text{out of the}\\ \text{system}\end{array}\right\}$$

where "flux" means "flow per unit time". Considering vehicular traffic flow as a conserved system, the analogous principle of conservation of vehicles can be applied to a fixed portion of roadway, and stated as follows:

$$\left\{\begin{array}{l}\text{the time rate of change}\\ \text{of vehicle concentration}\\ \text{of the roadway section}\end{array}\right\} = \left\{\begin{array}{c}\text{vehicle flow}\\ \text{into the}\\ \text{system}\end{array}\right\} - \left\{\begin{array}{c}\text{vehicle flow}\\ \text{out of the}\\ \text{system}\end{array}\right\} .$$

Drew (29, p. 307) expresses the principle of conservation of vehicles over a roadway section of length dx and for a time period of duration dt as

$$k\,dx - (k - \frac{\partial k}{\partial t}dt)dx = q\,dt - (q + \frac{\partial q}{\partial x}dx)dt \qquad 9$$

where:

$q$ = the flow into the roadway section at point $x$;

$q\,dt$ = the number of vehicle units entering the roadway section at point $x$ during time period $dt$;

$k$ = the concentration of the roadway section at time $t$;

$k\,dx$ = the number of vehicle units on the roadway section of length $dx$ at time $t$.

Considering the right-hand-side (R.H.S.) of Equation 9, the change in flow over the roadway section of length dx is denoted by $\partial q/\partial x$; if the net flow over the roadway section is defined as the flow into the section at point x minus the flow out of the section at point x + dx, then it follows that a positive (negative) change in flow is associated with a negative (positive) net flow. Similarly, considering the left-hand-side (L.H.S.) the change in concentration over the time period of duration dt is denoted by $\partial k/\partial t$; if the net concentration is defined as the concentration of the section at time t minus the concentration of the section at time t + dt, then it follows that a positive (negative) change in concentration is associated with a negative (positive) net concentration. And, considering R.H.S. and L.H.S. simultaneously, it follows that a positive (negative) net flow is associated with a positive (negative) change in concentration. The above is able to be summarized as follows:

$$\frac{\partial q}{\partial x} \gtrless 0 \leftrightarrow \text{net flow} \lessgtr 0,$$

$$\frac{\partial k}{\partial t} \gtrless 0 \leftrightarrow \text{net concentration} \lessgtr 0,$$

$$\text{net flow} \gtrless 0 \leftrightarrow \frac{\partial k}{\partial t} \gtrless 0.$$

That is, $\partial q/\partial x$ and $\partial k/\partial t$ are opposite in sign; reducing Equation 9,

$$k\ dx - (k - \frac{\partial k}{\partial t}\ dt)dx = q\ dt - (q + \frac{\partial q}{\partial x}\ dx)dt$$

$$k\ dt - k\ dx + \frac{\partial k}{\partial t}\ dt\ dx = q\ dt - q\ dt - \frac{\partial q}{\partial x}\ dx\ dt$$

$$\frac{\partial k}{\partial t}\ dt\ dx = - \frac{\partial q}{\partial x}\ dx\ dt$$

$$\frac{\partial k}{\partial t} = - \frac{\partial q}{\partial x}\ ,$$

and transposing,

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0\ . \qquad 10$$

Reconsidering R.H.S. of Equation 9, the term $q + \partial q/\partial x\ dx$ denotes the flow out of the roadway section at point $x + dx$, and is expressed as the flow into the section at point $x$ minus the net flow, denoted by $- \partial q/\partial x\ dx$. Analogously from L.H.S. of Equation 9, the term $k - \partial k/\partial t\ dt$ denotes the concentration of vehicle units on the roadway section at time $t + dt$, and is expressed as the concentration of the section at time t minus the net concentration, denoted by $\partial k/\partial t\, dt$.

Equation 10 is the "equation of continuity" for a one-dimensional, continuous, compressible system. A flow is termed "incompressible" if its concentration is not variable; otherwise, it is termed "compressible". Substituting Equation 1 into Equation 10,

$$\frac{\partial k}{\partial t} + \frac{\partial ku}{\partial x} = 0$$

or,

$$\frac{\partial k}{\partial t} + k\,\frac{\partial u}{\partial x} + u\,\frac{\partial k}{\partial x} = 0. \qquad 11$$

Greenberg's assumption (41, p. 80) that speed is "a function of density only", implies that

$$\frac{\partial u}{\partial t} = \frac{du}{dk}\,\frac{\partial k}{\partial t}\,, \qquad 12$$

$$\frac{\partial u}{\partial x} = \frac{du}{dk}\,\frac{\partial k}{\partial x}\,. \qquad 13$$

Substituting Equation 13 into Equation 11,

$$\frac{\partial k}{\partial t} + k\,\frac{du}{dk}\,\frac{\partial k}{\partial x} + u\,\frac{\partial k}{\partial x} = 0$$

or,

$$\frac{\partial k}{\partial t} + \frac{\partial k}{\partial x}\left(k\,\frac{du}{dk} + u\right) = 0. \qquad 14$$

The behavior of fluid motion is described by an "equation of motion". Drew (29, p. 308) employs the following equation of motion to describe one vehicle unit operating within a traffic stream:

$$\frac{du}{dt} = -c^2 k^n \frac{\partial k}{\partial x} \qquad 15$$

where:

$n$ = exponent of proportionality;

$c$ = constant determined from the state of traffic.

The term $du/dt$ denotes the acceleration of the traffic stream, while $\partial k/\partial x$ denotes the change in concentration over distance of the stream. The individual vehicle unit adjusts its speed as a function of the conditions of the traffic stream: the individual unit accelerates (decelerates), if traffic is becoming less (more) concentrated. For the system specified by these equations of continuity and motion, Greenberg (41, p. 81) has shown for $n = -1$ that

$$u = c \ln\left(\frac{k_j}{k}\right). \qquad 16$$

The general result for $n > -1$ has been demonstrated (29, p. 309) to be

$$u = u_f\left[1-\left(\frac{k}{k_j}\right)^{(n+1)/2}\right]; \qquad 17$$

substituting Equation 1 into Equation 17,

$$q = ku_f[1-(\frac{k}{k_j})^{(n+1)/2}]; \qquad 18$$

finally, substituting Equation 1 into Equation 18,

$$k = k_j(1 - \frac{u}{u_f})^{2/(n+1)} . \qquad 19$$

Traffic flow, concentration, and speed determine the vehicular traffic "state"; obviously, the three are not independent: the values of any two determine the value of the third. An equation -- such as Equation 1, 17, 18, and 19 -- which displays these principles is termed an "equation of state".

The equations of state allow one to solve for the theoretical maximum flow, $q_m$, and for the levels of concentration and speed consistent with that flow. Taking the derivative of Equation 18 with respect to concentration

$$\frac{dq}{dk} = u_f[1 - \frac{n+3}{2}(\frac{k}{k_j})^{(n+1)/2}]$$

and setting it equal to zero

$$u_f[1 - \frac{n+3}{2}(\frac{k_m}{k_j})^{(n+1)/2}] = 0$$

yields the concentration, $k_m$, associated with maximum flow

$$k_m = [2/(n+3)]^{2/(n+1)} k_j; \qquad 20$$

substituting Equation 20 into Equation 17 results in

$$u_m = [(n+1)/(n+3)]u_f; \qquad 21$$

multiplying Equation 20 by Equation 21 yields

$$q_m = [2/(n+3)]^{2/(n+1)}[(n+1)/(n+3)]k_j u_f \;. \qquad 22$$

In view of the precedent established by theoretical and empirical considerations cited in a previous section, these results are now expressed in a form compatible with the Greenshields linearity assumption, by making the substitution $n = 1$ in Equations 17-22; this substitution produces, respectively, the following set (a subset of which appears as Equations 3-7) of equations:

$$u = u_f(1 - \frac{k}{k_j}) \qquad 23$$

$$q = ku_f(1 - \frac{k}{k_j}) \qquad 24$$

$$k = k_j(1 - \frac{u}{u_f}) \qquad 25$$

$$k_m = \frac{1}{2} k_j \qquad 26$$

$$u_m = \frac{1}{2} u_f \qquad 27$$

$$q_m = \frac{1}{4} k_j u_f \;. \qquad 28$$

<u>The fluid-energy analogy -- acceleration noise</u>

Whereas the previous section analyzed the properties of the traffic stream by appealing to the fluid analogy applied to that stream in a macroscopic manner, the present section analyzes those and other properties by appealing to a fluid-energy analogy, applied to the traffic stream in the microscopic sense.

The principle of "conservation of energy" states that the total energy of a control system can be neither created nor destroyed, although "it may appear in several forms... and it may be transformed from one type to another" (29, p. 368). The present section is concerned with one-dimensional, compressible flow; in such an analysis of compressible flow, it is useful to express the principle of conservation of energy in the form of the total energy equation (153, p. 393)

$$\Omega = e + \frac{mv^2}{2} + \phi$$

where:

$\Omega$ = total energy per unit fluid mass;

$e$ = internal energy;

$\frac{mv^2}{2}$ = kinetic energy;

$\phi$ = potential energy.

Given a unit fluid mass characterized by some condition of state and at rest, one is able to change its state and induce motion by applying a quantity of heat and force from an external medium; the internal energy of the mass is defined to be the algebraic sum of the quantity of heat absorbed by the fluid plus the force imposed upon the fluid minus the accompanying increase of the kinetic energy of the motion acquired by the fluid. The kinetic energy of a unit of fluid mass, m, is its ability to apply force as a result of its motion of velocity, v; quantitatively, it is the amount of force applied by the mass upon other masses as it is brought to rest (117). Finally, the potential energy of a unit fluid mass is the capacity of the mass for applying force by virtue of its position. In situations of high-velocity fluid flow, the potential energy of the mass is small relative to the internal and kinetic energy, and is omitted from consideration (113, p. 276). Thus, the total energy equation becomes

$$\Omega = e + \frac{mv^2}{2} .$$

Now, applying the analogy (14) between the flow of fluid and the flow of vehicular traffic, the internal energy of the traffic stream is manifest as dissipated or erratic vehicular motion, due to roadway geometrics and to the interaction of vehicle units. On the other hand, the kinetic energy of the traffic stream is the energy of motion of the stream; the mass of the fluid is conceived to be the concentration of traffic, k, while the velocity of the fluid becomes the speed of traffic, u. It is necessary to introduce (131) a kinetic-energy correction factor, $\alpha$, to account for the fact that the average speed of the traffic stream at any point along the roadway section is not simply the average of the speeds of the vehicle units passing that point; for, while the latter average may vary from point to point, the former does not: that is, the speed of vehicle units may vary within the traffic stream. The traffic total energy equation thus becomes

$$\Omega = e + \alpha\ ku^2;$$

substituting Equation 1 into this result,

$$\Omega = e + \alpha\ qu\ .$$

In a previous section it was noted that acceleration noise is a measure of the smoothness of flow of the traffic stream; within the context of the present analogy, therefore, acceleration noise represents the internal energy of the traffic stream. Therefore,

$$\Omega = \sigma + \alpha\ qu; \qquad 30$$

energy is not lost from the system, but merely is converted from one form to another. Drew summarizes the argument as follows (29, p. 369):

One recalls from the second law of thermodynamics that the mechanical forms of energy, such as kinetic energy, are more valuable than an equivalent amount of thermal energy or internal energy. This is certainly true in the case of traffic flow. Thus one can say that the forces of friction (adverse geometrics and traffic interaction) tend to convert the desirable forms of energy (traffic motion) into the less valuable forms (traffic interaction).

The previous section solved for the levels of concentration and speed consistent with the theoretical maximum flow. Similarly (14), one is able to solve for the levels of flow, concentration and speed -- $q_m'$, $k_m'$, and $u_m'$, respectively -- consistent with maximum kinetic energy. Squaring Equation 18

$$q^2 = k^2u_f^2[1 - (\frac{k}{k_j})^{(n+1)/2}]^2$$

and dividing by k

$$\frac{q^2}{k} = ku_f^2[1 - 2(\frac{k}{k_j})^{(n+1)/2} + (\frac{k}{k_j})^{(n+1)}].$$

If E denotes the expression of kinetic energy, as it appears in Equation 30, then the division of the last result by E gives

$$\frac{q^2}{(kE)} = ku_f^2[1 - 2(\frac{k}{k_j})^{(n+1)/2} + (\frac{k}{k_j})^{(n+1)}]/E$$

or

$$\frac{q^2}{(k\alpha qu)} = ku_f^2[1 - 2(\frac{k}{k_j})^{(n+1)/2} + (\frac{k}{k_j})^{(n+1)}]/E;$$

solving for E,

$$E = \alpha k u_f^2 [1 - 2(\frac{k}{k_j})^{(n+1)/2} + (\frac{k}{k_j})^{(n+1)}]$$

or

$$E = \alpha k u_f^2 - 2\alpha u_f^2 \frac{k^{(n+3)/2}}{k_j^{(n+1)/2}} + \alpha u_f^2 \frac{k^{(n+2)}}{k_j^{(n+1)}} \, .$$

Taking the partial derivative of this result with respect to concentration yields

$$\frac{\partial E}{\partial k} = \alpha u_f^2 - (n+3)\ \alpha u_f^2 (\frac{k}{k_j})^{(n+1)/2} + (n+2)\ \alpha u_f^2 (\frac{k}{k_j})^{(n+1)}$$

and setting it equal to zero

$$\alpha u_f^2 - (n+3)\ \alpha u_f^2 (\frac{k'_m}{k_j})^{(n+1)/2} + (n+2)\ \alpha u_f^2 (\frac{k'_m}{k_j})^{(n+1)} = 0$$

or

$$1 - (n+3)(\frac{k'_m}{k_j})^{(n+1)/2} + (n+2)(\frac{k'_m}{k_j})^{(n+1)} = 0;$$

this expression has the quadratic form

$$ax^2 + bx + c = 0$$

where:

$$x = (\frac{k'_m}{k_j})^{(n+1)/2} ;$$

$a = n + 2;$

$b = -(n+3);$

$c = 1.$

The roots are 1 and 1/(n+2); the former is consistent with $u = q = 0$ and minimizes the quadratic; the latter root maximizes the quadratic and implies that

$$\left(\frac{k'_m}{k_j}\right)^{(n+1)/2} = \frac{1}{(n+2)}$$

or

$$k'_m = k_j\left[\frac{1}{(n+2)}\right]^{2/(n+1)} . \qquad 31$$

Substituting Equation 31 into Equation 17 produces

$$u'_m = u_f\left[1 - \left(\frac{k_j\left(\frac{1}{n+2}\right)^{2/(n+1)}}{k_j}\right)^{(n+1)/2}\right]$$

or

$$u'_m = u_f\ \frac{(n+1)}{(n+2)} . \qquad 32$$

Substituting Equations 31 and 32 into Equation 1 yields the result that

$$q'_m = \left[k_j\left(\frac{1}{n+2}\right)^{2/(n+1)}\right]\left[u_f\ \frac{(n+1)}{(n+2)}\right]$$

or

$$q'_m = k_j u_f\ \frac{n+1}{(n+2)^{(n+3)/(n+1)}} . \qquad 33$$

These, then, are the levels of flow, concentration, and speed associated with maximum kinetic energy of the traffic stream.

One is able to consider two boundary conditions of the traffic total energy equation (29). Equation 30

$$\Omega = \sigma + \alpha qu$$

exhibits these conditions; on the one hand, as acceleration noise approaches zero, kinetic energy approaches total energy; that is,

$$\Omega = \alpha qu.$$

Under the condition of maximum kinetic energy,

$$\Omega = \alpha q'_m u'_m$$

or

$$\Omega = \alpha k_j u_f^2 \frac{(n+1)^2}{(n+2)^{2(n+2)/(n+1)}} .$$

On the other hand, the second boundary condition states that as kinetic energy approaches zero, acceleration noise approaches a maximum; that is

$$\Omega = \sigma_{max} .$$

Equating the two previous results,

$$\sigma_{max} = \alpha k_j u_f^2 \frac{(n+1)^2}{(n+1)^{2(n+2)/(n+1)}} ;$$

solving for the kinetic-energy correction factor,

$$\alpha = \frac{\sigma_{max}}{k_j u_f^2} \frac{(n+2)^{2(n+2)/(n+1)}}{(n+1)^2} .$$

Equation 8 defined acceleration noise; assuming acceleration noise to be a function of speed, one can deduce the form of the functional relationship by substituting the second boundary condition into Equation 30 to derive

$$\sigma = \sigma_{max} - \alpha q u$$

or

$$\sigma = \sigma_{max} - \alpha k u^2.$$

Substituting Equation 19 into this result yields

$$\sigma = \sigma_{max} - \alpha [k_j (1 - \frac{u}{u_f})^{2/(n+1)}] u^2$$

or

$$\sigma = \sigma_{max} - \sigma_{max} \frac{(n+2)^{2(n+2)/(n+1)}}{(n+1)^2} (\frac{u}{u_f})^2 (1 - \frac{u}{u_f})^{2/(n+1)} . \qquad 34$$

Equation 34 is subject to the following boundary conditions:

$\sigma = \sigma_{max}$ at $u = 0$, and

$\sigma = \sigma_{max}$ at $u = u_f$.

To express the main results of this section in a form compatible with the Greenshields linearity assumption, one makes the substitution $n = 1$ in Equations 31-34 to produce, respectively, the following set of equations:

$$k_m' = \frac{1}{3} k_j \qquad 35$$

$$u_m' = \frac{2}{3} u_f \qquad 36$$

$$q_m' = \frac{2}{9} k_j u_f \qquad 37$$

$$\sigma = \sigma_{max} - \frac{27}{4} \sigma_{max} [(\frac{u}{u_f})^2 - (\frac{u}{u_f})^3] . \qquad 38$$

Summary

The fluid analogy serves as an efficient device to describe the properties of vehicular traffic by the application of certain principles of fluid mechanics. In order to completely establish the relationship between the speed-acceleration noise model and the fundamental diagram, one is, now, in a position to relate the levels of flow, concentration, and speed consistent with the optimization of the vehicular traffic stream from the point of view of these two models. Equating Equations 26 and 35, Equations 27 and 36, and Equations 28 and 37, respectively, one obtains the following results:

$$k'_m = \frac{2}{3} k_m , \qquad 39$$

$$u'_m = \frac{4}{3} u_m , \qquad 40$$

and

$$q'_m = \frac{8}{9} q_m . \qquad 41$$

Appealing solely to the implications of the fundamental diagram, one would conclude that optimal use of an urban roadway section requires levels of concentration and speed consistent with maximum flow, $q_m$. However, if the object of urban roadway is to provide maximum level of service, acceleration noise must be minimized. "In a traffic system, this concept of efficiency is manifest by maximizing the kinetic energy of the stream as a whole and minimizing the acceleration noise of the individual vehicles (internal energy)" (29, p. 383). Maximizing level of service results in concentration and speed superior to those associated with maximum flow, $q_m$; the trade-off is the diminution of flow to $q_m'$: a smaller number of vehicle units per unit time enjoys an enhanced level of service upon the roadway section.

## The Basic Model -- A Diagrammatic Approach

Having reviewed the mathematical derivations of, and relationships between, the fundamental diagram of road traffic and the speed-acceleration noise model, one is in a position to consider the apparatus in diagrammatic form. This consideration will preview the direction of statistical estimation of the following chapter.

Walters (152) related the "time of a trip-mile" to vehicular traffic concentration and to vehicular traffic flow. His approach was formalized by Johnson (68); defining "time" to be the reciprocal of speed, Johnson demonstrated that the theoretical maximum flow, $q_m$, on the flow-time function depicted in Figure 5 is consistent with concentration $k_m$ on the concentration-time function, and that this level of concentration is defined by the point of tangency between the

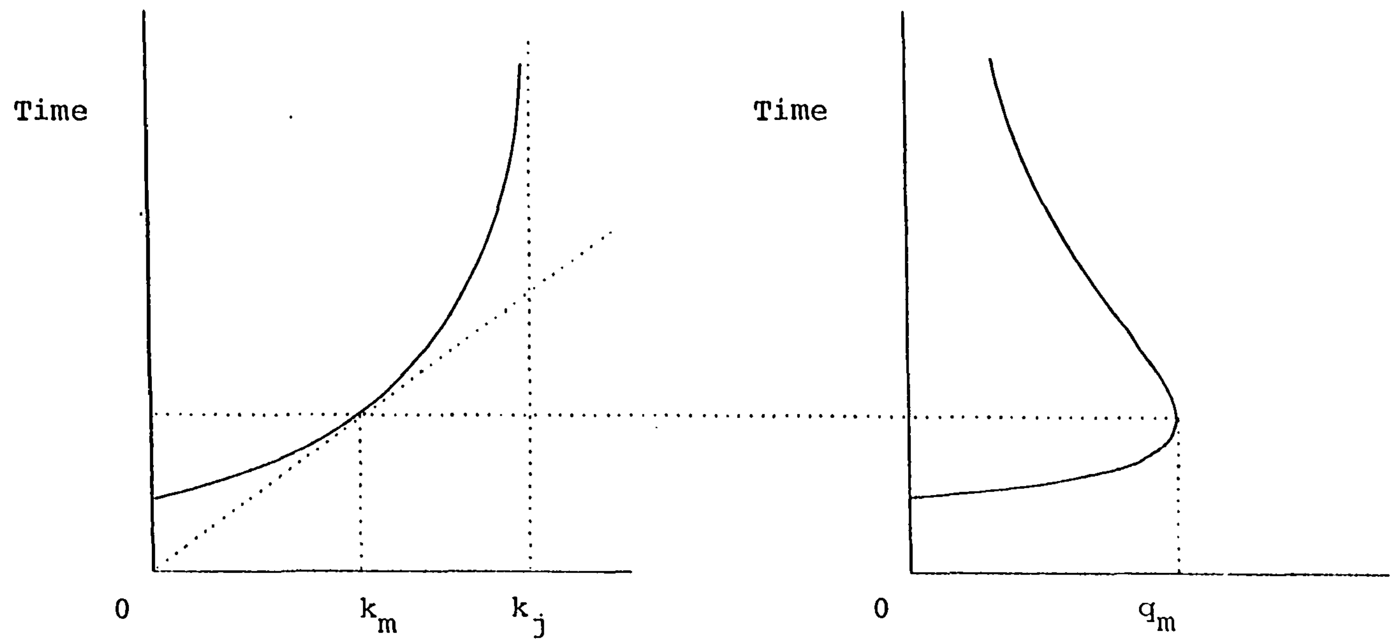


Figure 5. The flow-concentration-time relationship

concentration-time function and a ray through the origin. The level of concentration $k_j$ is the absolute maximum concentration. As concentration approaches jam concentration, time approaches infinity; simultaneously, the flow-time function asymptotically approaches the time axis. These functions manifest the principle that a given level of vehicular traffic flow can occur, accompanied by either a level of concentration greater than $k_m$ (as associated with the negatively-sloped segment of the flow-time function) or by a level less than $k_m$ (as associated with the positively-sloped segment of the flow-time function): only at the level of flow $q_m$ is there a one-to-one correspondence between flow and concentration.

Recalling that time is the reciprocal of speed, the consistency between the Walters-Johnson approach and the fundamental diagram of road traffic is evident. Figure 6 exhibits this consistency by synthesizing Figures 3 and 5. The Walters-Johnson flow-time function occupies the northwest quadrant, while the concentration-time function occupies the southwest quadrant; these two functions are simply a restatement of the fundamental diagram, occupying the northeast quadrant. Figure 6 suggests that Equation 1 is able to be expressed in the following form:

$$q = kt^{-1}, \qquad 42$$

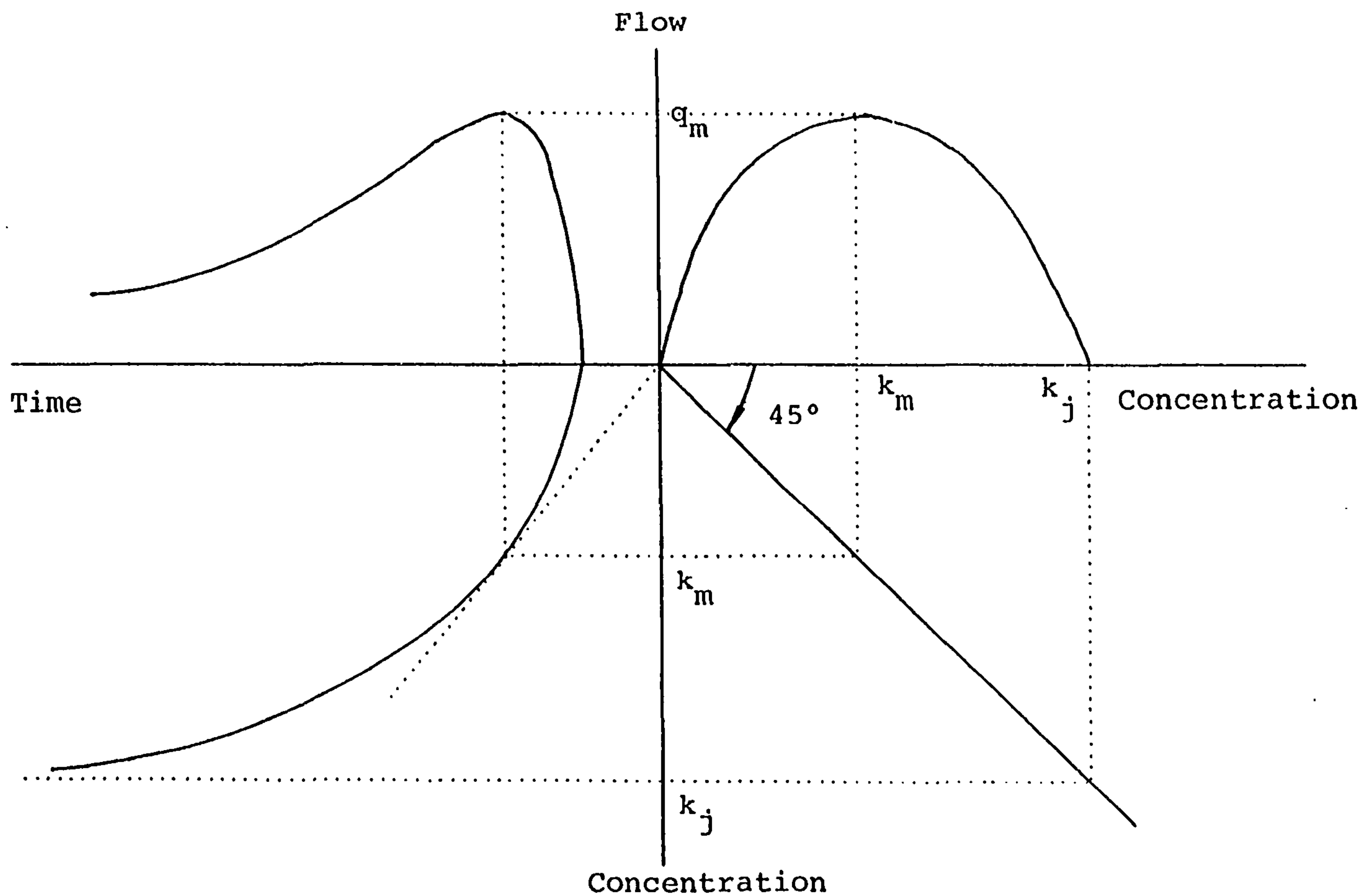


Figure 6. The fundamental diagram and the flow-concentration-time relationship

such that

$$t = t(q) \qquad 43$$

or

$$t = t(k) \qquad 44$$

where

$t$ = time.

That is, Equation 43 applies to the northwest quadrant of Figure 6, while Equation 44 applies to the southwest quadrant.

Focusing consideration, now, upon the speed-acceleration noise model, Equation 38 was derived under the assumption that the acceleration noise of a vehicle unit is a function of the speed of the traffic system within which the vehicle unit is operating. When operating upon an urban roadway section with traffic flow at a level which does not impede maneuverability, the vehicle operator characteristically attempts to maintain a uniform rate of speed. Typically, though, the vehicle is subject to inadvertent deviations from uniform speed. Moreover, when traffic flow increases to a level at which interaction among vehicle units is present and the individual vehicle is unable to easily maintain its desired speed, the vehicle deviates from a uniform rate of speed to a greater extent by changing lanes, passing slower-moving vehicles, and the like.

That is, "total measured acceleration noise" is composed of two components: "natural noise", attributed to the geometrics of the roadway segment and to the behavior of the vehicle operator; and "interaction noise", attributed to vehicle interaction. The speed-acceleration noise model is shown in Figure 7 (29, p. 371); $\sigma_N$ denotes the natural noise, $\sigma_I$ denotes the interaction noise, and the sum of these two, the total measured noise, is denoted by $\sigma$. As speed approaches the levels $u = 0$ and $u = u_f$, acceleration noise approaches the maximum, $\sigma_{max}$. At the level of speed consistent with maximum kinetic energy, interaction noise is zero and acceleration noise is minimized: at $u = u_m'$, $\sigma_I = 0$ and $\sigma = \sigma_N = \sigma_{min}$. Equation 38 is the functional form of Figure 6.

The relationship between the fundamental diagram and the speed-acceleration noise model is presented as Figure 8, in which Figure 6 (rotated 90° clockwise about its origin, and 180° about the resultant vertical axis) and Figure 7 (the speed-acceleration noise model has been transformed to a time-acceleration noise model) are synthesized. Figure 8 is the diagrammatic equivalent of the mathematical relationships between the fundamental diagram and the speed-acceleration noise model, derived under the analogy of vehicular traffic flow and fluid flow. The levels of time, $t_m$ and $t_m'$, are consistent with the levels of speed, $u_m$ and $u_m'$,

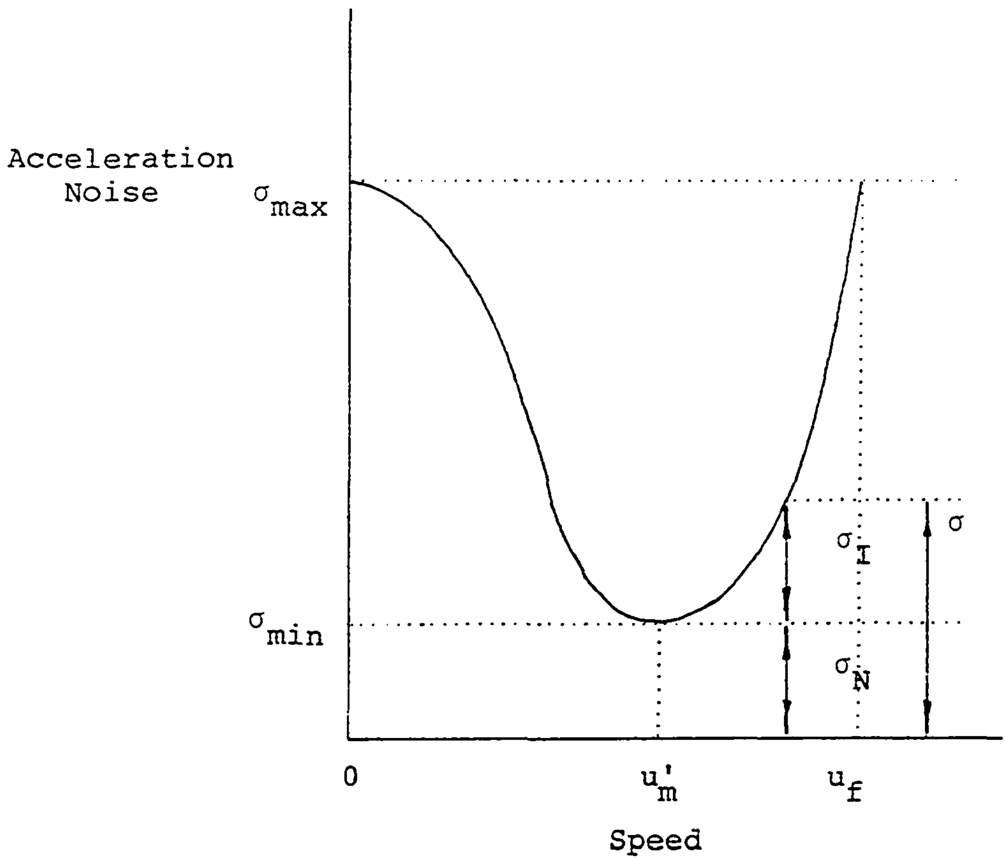


Figure 7. The speed-acceleration noise relationship

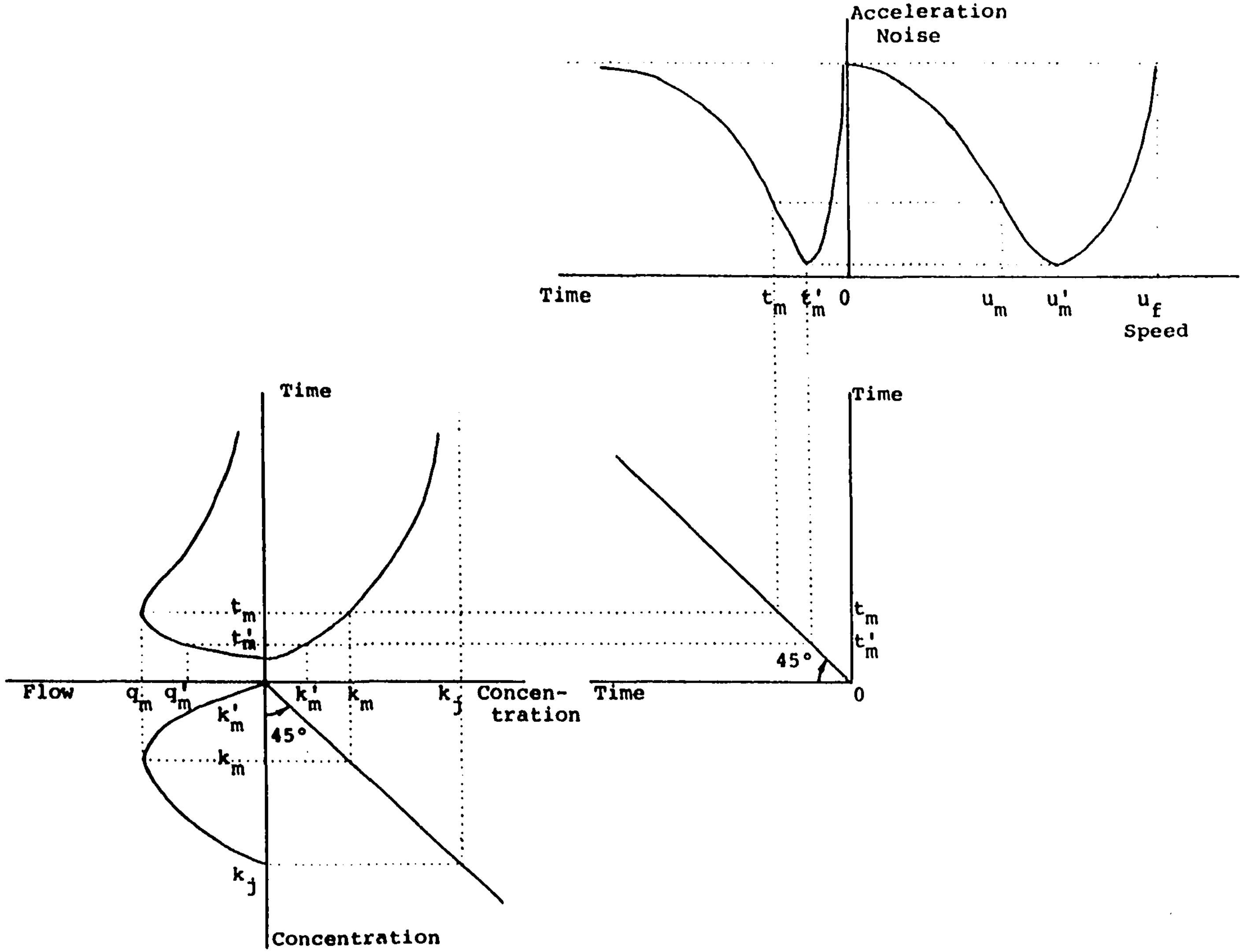


Figure 8. The relationship between the fundamental diagram and the speed-acceleration noise model

respectively.

Greenshields examined the relationship between his quality index, Q, and speed, and between his index and gasoline economy. He observes (43, p. 32) that the "direct correlation between speed and the quality index and between fuel consumption and the index leads to the question of why speed or fuel consumption cannot be used in place of the quality index.... Further testing will be needed to demonstrate stability (or lack of it) in the relationship between speed or fuel consumption and the quality index."

Appealing to the fluid-energy analogy, Capelle reasons that there should be a relationship between the internal energy of the vehicular traffic stream and fuel consumption. He assumes (14, p. 84) that "the acceleration noise measured over a segment of roadway is equal to the total fuel consumed, F.C., minus the minimum fuel consumption, $F.C._{min}$." Substitution of this assumption

$$\sigma = F.C. - F.C._{min}$$

into Equation 38 leads to Capelle's proposed model

$$F.C. = F.C._{max} - \frac{27}{4}(F.C._{max} - F.C._{min})\left[\left(\frac{u}{u_f}\right)^2 - \left(\frac{u}{u_f}\right)^3\right];$$

Capelle fails to rigorously define $F.C._{max}$ and $F.C._{min}$. Drew (29, p. 384) favorably comments on the proposed model:

> This expression for fuel consumption as a function of speed appears to be realistic. For example, it is an accepted fact that the operational costs of a vehicle driven at high speeds under free-flow conditions are considerably more than those experienced when driving at a reasonable speed under free-flow conditions. It is also realistic to expect operational costs to increase at low constrained speeds when a motorist is subjected to stop-and-go conditions.

Presuming that the Capelle hypothesis is correct, other vehicle operating costs -- namely, oil consumption, tire wear, and the like -- can be assumed to embrace the same relationship with acceleration noise, as does fuel consumption.

The present approach is to make the following, more straightforward and comprehensive assumption: the acceleration noise of a vehicle unit is proportional to the vehicle unit operating cost; that is,

$$\sigma = a \; O.C.,$$

where:

$a > 0$;

O.C. = operating cost of a vehicle unit.

Substitution of this transformation into Equation 38 produces

$$O.C. = O.C._{max} - \frac{27}{4} O.C._{max} \left[ \left(\frac{u}{u_f}\right)^2 - \left(\frac{u}{u_f}\right)^3 \right] . \qquad 45$$

Figure 9, the basic model, reflects this transformation. The flow-time function is labeled ASTC, average social time cost; and, the speed-operating cost function is labeled ASOC, average social operating cost. These cost functions are

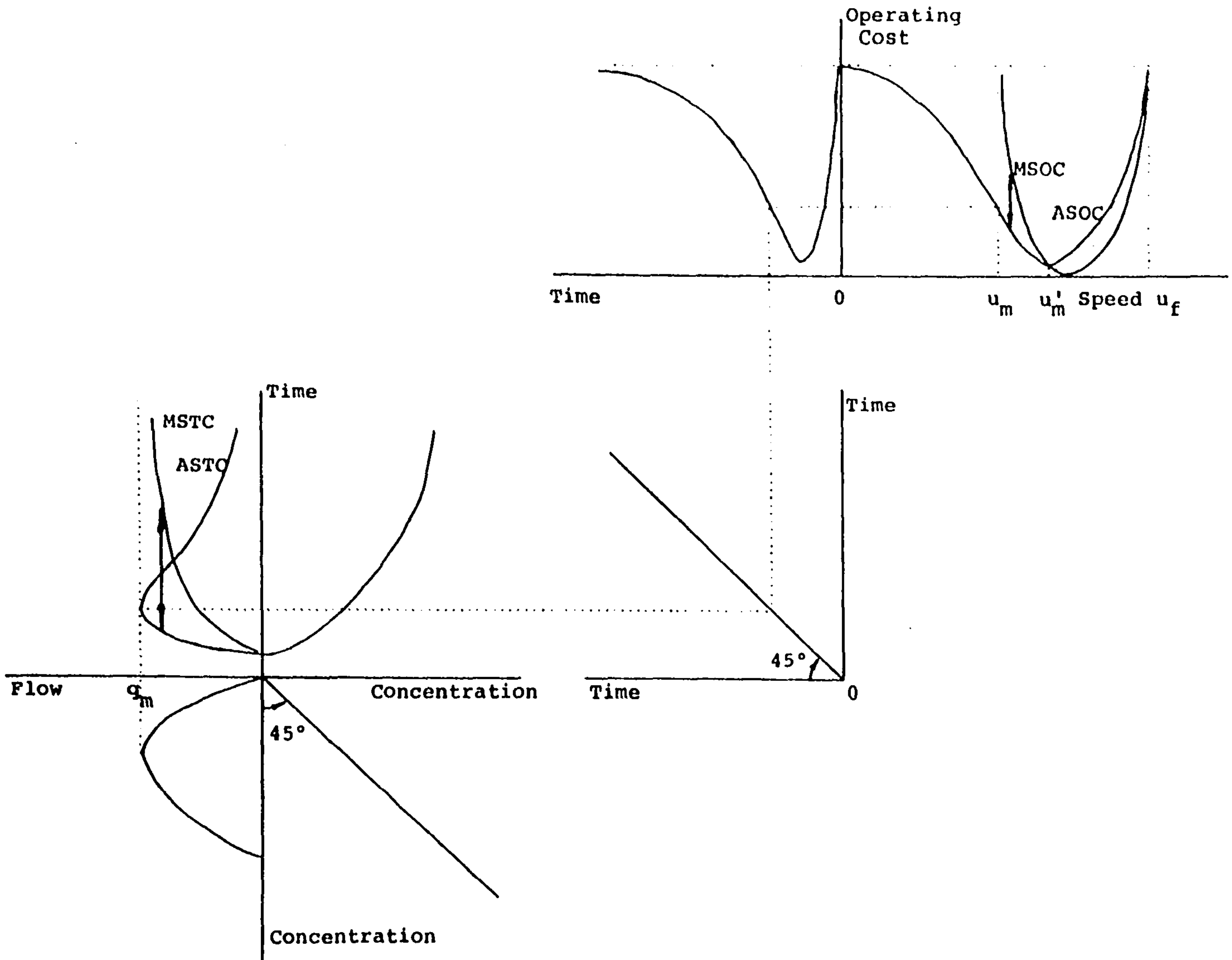


Figure 9. The basic model

analogous to the SRAC function of Figure 2. The functions MSTC and MSOC are the curves marginal to the ASTC and ASOC curves, respectively, under an assumed dependence of unit transportation cost upon system flow; Walters (150) demonstrated the relationship between an average and a marginal cost curve. Applying his method to the flow-time function, let t(q) denote the time cost to a vehicle unit operating on a roadway section, when flow is at the level q. Then, the change in time cost generated by a change in flow on the roadway section is given by

$$MSTC = \frac{d}{dq} q \cdot t(q)$$

$$= t(q) + q \cdot \frac{d}{dq} t(q)$$

$$= t(q) \; [1 + \frac{q}{t(q)} \cdot \frac{d}{dq} t(q)]$$

$$= t(q) \; (1+\varepsilon)$$

where:

$\varepsilon$ = elasticity of ASTC.

That is,

$$MSTC = ASTC \; (1+\varepsilon_{ASTC}). \qquad 46$$

In this derivation, the equation

$$MSTC = t(q) + q \cdot \frac{d}{dq} t(q)$$

expresses the principle that the sum of the average social time cost plus the optimal toll equals the marginal social time cost at flow q.

In the present analysis -- assuming unit transportation cost to be a function of system flow -- the time cost component of the optimal urban roadway toll is to be estimated from the flow-time function (based upon the fundamental diagram of road traffic), while the operating cost component is to be estimated from the flow-operating cost function (based upon the speed-acceleration noise function). These two components are, diagrammatically, the vertical distance between the ASTC and MSTC, and between the ASOC and MSOC, respectively, as shown in Figure 9.

# CHAPTER V: IMPLEMENTATION OF THE MODEL

## Introduction

The estimation of a roadway price schedule requires, on the one hand, knowledge of the state of the vehicular traffic system and, on the other, knowledge of the costs incurred by the individual vehicle unit as it operates within a traffic system characterized by a particular state. Roth (109, p. 310) observes, relative to the former requirement, that the "relationship traffic volumes and speeds is best found by experiment in the road networks being investigated." A similar observation applies to the latter requirement as well.

The present analysis approaches satisfaction of these requirements by application of the linear (that is, with n = 1) model depicted in Figure 9. With respect to the former requirement, the data under consideration has been supplied by the Texas Transportation Institute, Urban Transportation Systems Program, of Texas A&M University. With respect to the latter requirement, the time cost of the individual vehicle unit is to be estimated by means of the standard procedure pioneered by A. A. Walters; and, the operating cost is to be estimated by applying this standard procedure to the speed-acceleration noise model, with the spirit of enhancing the administrative feasibility of instituting urban roadway pricing.

## Urban Roadway Cost Functions

### Introduction

Theoretical arguments have been delivered relative to the shape of cost curves, and statistical enquiry has favored the conclusions of certain arguments at the expense of others (151). Johnston notes that the statistical approach to the cost function requires data which satisfies four general conditions (71, p. 26):

> 1. The basic time period for each pair of observations should be one in which the observed output was achieved by a uniform rate of production within the period.... 2. The observations on cost and output should be properly paired in the sense that the cost figure is directly associated with the output figure.... 3. We should also like a wide spread of output observations so that cost behavior could be observed at widely differing rates of output.... 4. It would also be necessary to keep the experimental data uncontaminated by the influence of factors extraneous to the cost-output relationship itself.... The four requirements above have been stated with reference to the ideal data for testing short-run cost-output relationships.

The extent to which these requirements are satisfied in the present analysis will become apparent; let it be mentioned at this point, however, that a conspicuous inadequacy of the data utilized here pertains to the third requirement.

A threshold issue of the present analysis relates to the traditional procedure employed to estimate the operating cost component of roadway price schedules. On the topic of statistical cost functions, Meyer comments as follows (89, p. 212):

> Finally, it always must be borne in mind that statistical costs are inevitably based on historical information. This is usually true of cost estimates, of course, but it is always the case with statistical estimates. Being historical, statistical costs must be used cautiously as guides to decisions about the future, since the important fact about the future often could be that the circumstances producing the historical cost experience are or should be altered.

And, again, Walters (151, p. 46) notes that "short run cost theory is framed in terms of a unit economic period whereas accounting data are usually collected for longer periods." Invariably, the operating cost of a vehicle unit maneuvering on a roadway section is assessed from historical data: whereas, the present approach, first of all, is based upon the contention that the appropriate data is "real-time", rather than historical; and, secondly, by employing the speed-operating cost model depicted in Figure 9, is more administratively and theoretically tractable than the traditional procedure.

In the ideal situation, furthermore, all cost components required to establish a set of roadway prices would be monitored by a "closed loop system" (58, p. 3):

> In this context, a closed loop system is one that provides for surveillance of traffic operations, acquiring data on those operations which can be processed by a computational logic in a real-time computer, testing observed conditions against a set of decision rules, selecting commands in light of the results of the tests,

> activating appropriate controls or communicating with drivers to improve traffic movement when necessary, and then re-assessing the traffic behavior to determine if further corrections are to be made.

Such a system would allow prices to be established, "taking into account highway geometry and the environmental conditions, rather than averaged historical data which may not apply to the situation at a particular time" (58, p. 14); it would facilitate implementation of Smeed's off-vehicle recording system of point pricing (123, p. 18): that is, the levying of an appropriate price upon the vehicle unit as it passes strategic points along the roadway section, the recording of the assessed charge at a central computer (rather than within the vehicle unit, itself), and the billing of the vehicle operator in a way analogous to the billing of customers of telephone service.

The linear model depicted in Figure 9 would provide such a closed loop system with the capacity to calculate roadway prices in a manner requiring a minimum amount of real-time data (31, p. 71; 58, p. 27; 60, p. 7). For, once the parameters of the model had been estimated for a particular roadway section, then the monitoring of any two of the vehicular traffic variables -- $q$, $k$, $u$, or $\sigma$ -- would allow computation of the optimal price in real time.

Study location

The north Central Expressway, Dallas, Texas, is a six-lane divided facility; a portion of the expressway is shown in Figure 10 (154). Figure 11 (30) delineates the Expressway section for which this study had access to data: namely, the three inbound lanes between the Mockingbird on-ramp and the Fitzhugh off-ramp. The section is approximately one and one-third miles in length.

The data requirements are twofold: macroscopic data, from which is generated the concentration-time function shown in Figure 9; and, microscopic data, from which is generated the speed-operating cost function shown in the figure. Both classes of data have been supplied by the Texas Transportation Institute.

The macroscopic data was derived from an "input-output" evaluation, continuously conducted by the Institute during the morning and afternoon periods of peak facility usage. The North Central Expressway is stratified into macroscopic "subsystems", by virtue of the location of automatic vehicle detectors. Three of these subsystems are located within the roadway section depicted in Figure 11, and are delineated as follows: from north of Mockingbird to the McCommas off-ramp, from the McCommas off-ramp to the Henderson off-ramp, and from the Henderson off-ramp to the Fitzhugh off-ramp. For the purpose of the present analysis, two of the vehicular

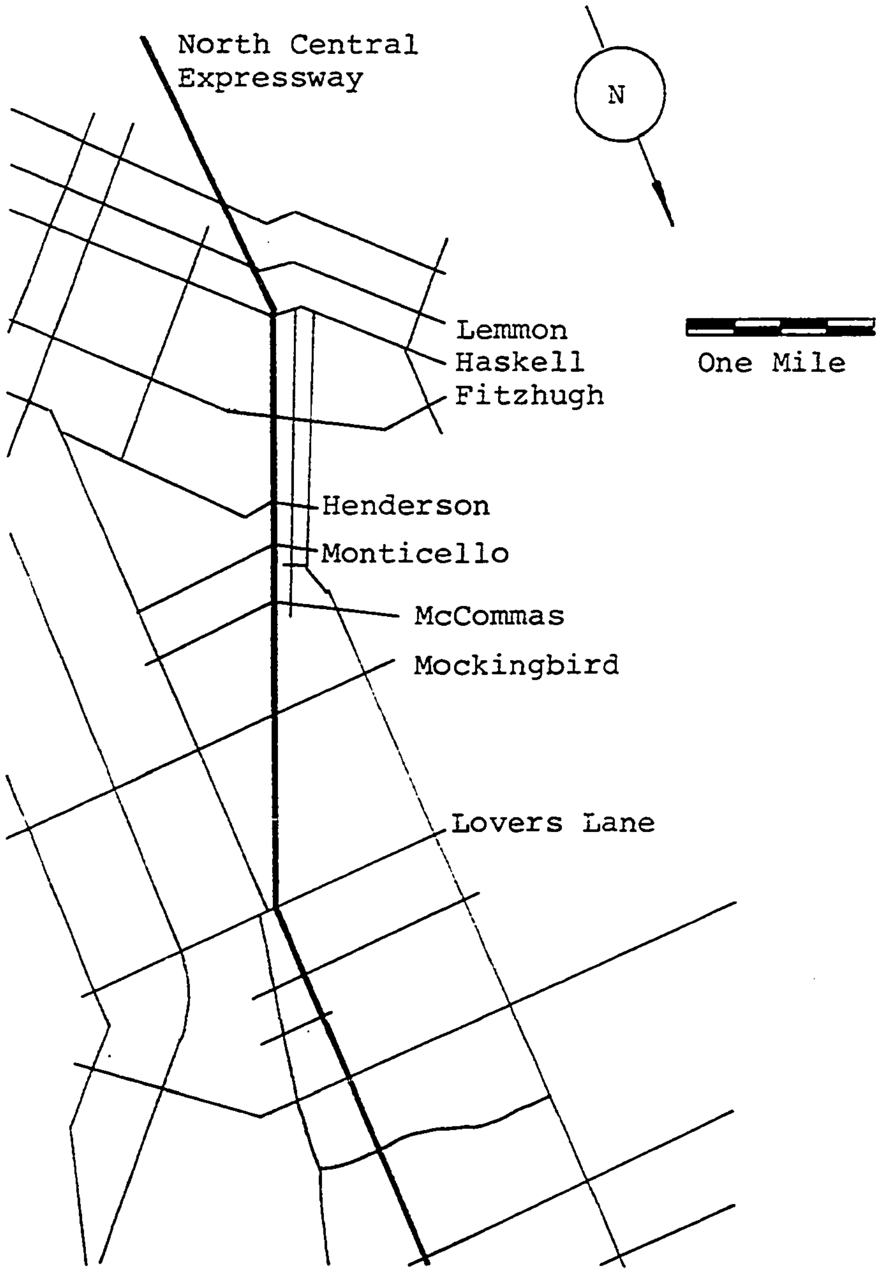


Figure 10. The North Central Expressway: Dallas, Texas

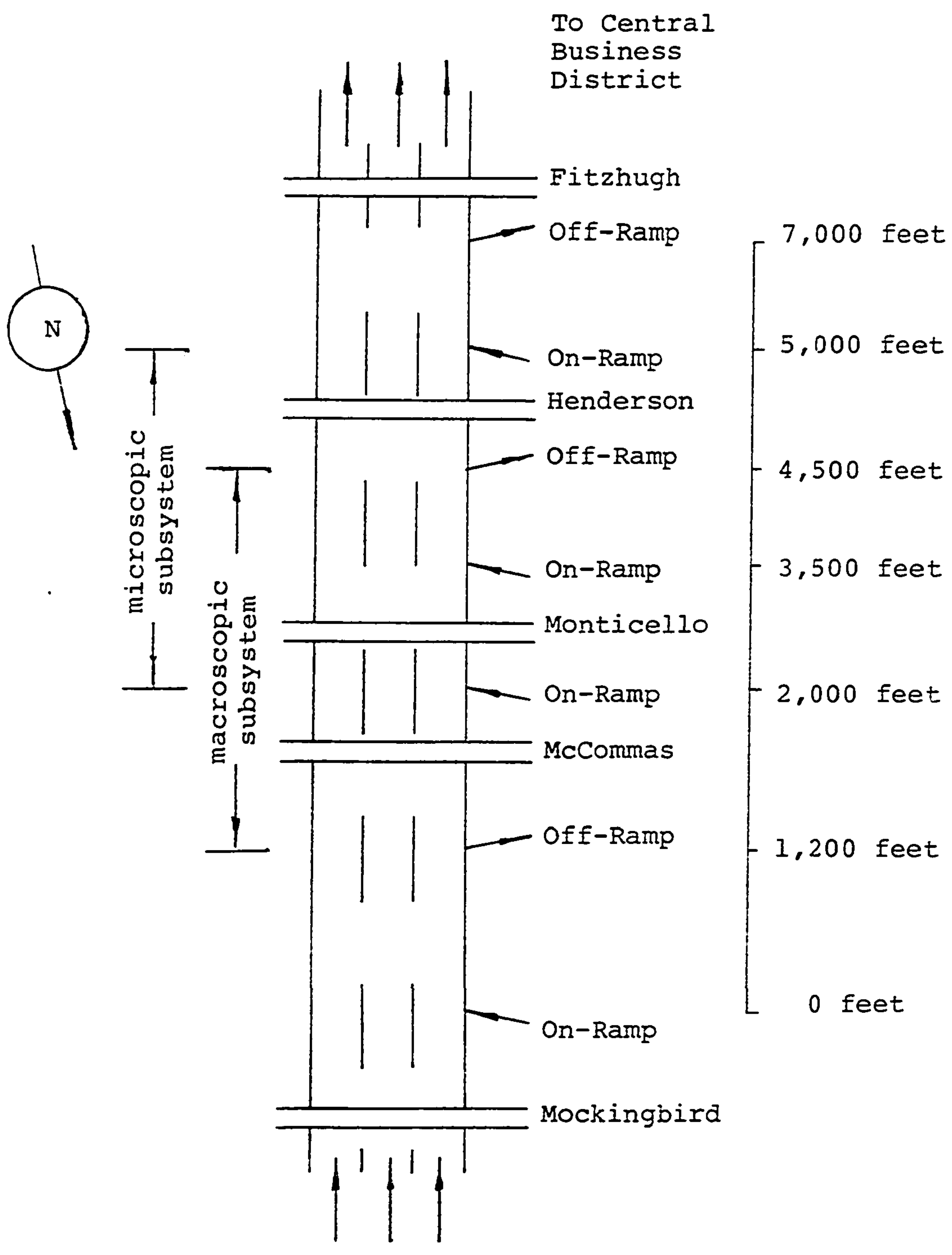


Figure 11. Study area of the North Central Expressway

traffic variables generated by the Institute for a particular macroscopic subsystem are of interest: namely, "density" (determined by the input-output study by monitoring flow into and flow out of the subsystem) and "calculated speed" (computed by the input-output study, such that it is a system measure, reflecting conditions in all three lanes). These two variables are simply k and u, respectively.

The microscopic data was derived from a "moving vehicle study" conducted during periods of peak facility usage; such a study injects into a vehicular traffic system an instrumented automobile, the operator of which is instructed to maneuver the vehicle as a "floating car" -- that is, to drive as he normally would, maintaining the pace of the traffic. This moving vehicle study stratifies the North Central Expressway into microscopic subsystems by the position of on-ramps; three of these subsystems are located within the roadway section depicted in Figure 11, and are described as follows: the Mockingbird on-ramp to the McCommas on-ramp, the McCommas on-ramp to the Monticello on-ramp, and the Monticello on-ramp to the Henderson on-ramp. For the purpose of the present analysis, the three vehicular traffic variables of interest generated by the Institute for a particular microscopic subsystem are the following: "mean velocity", the variable u; "acceleration noise", the variable

$\sigma$; and, "fuel consumption", the variable F.C.

Prior to data tabulation, the judgment was made to analyze data from a minimal number of subsystems, such that the chosen input-output study subsystem and the chosen moving vehicle study subsystem spatially coincided. Since the patterns of subsystems defined by the two studies are spatially incongruous, it was decided to employ macroscopic data from the subsystem defined by the McCommas off-ramp and the Henderson off-ramp, and to employ microscopic data from the subsystem defined by the McCommas on-ramp and the Henderson on-ramp; the latter subsystem, so specified, is the aggregate of two microscopic subsystems delineated by the moving vehicle study. Figure 11 depicts the subsystems under consideration. The model presented in Figure 9 is to be applied to the urban roadway section which constitutes these two subsystems: that is, the roadway section defined by the McCommas off-ramp and the Henderson on-ramp, approximately three-quarters of one mile in length.

Input-output study data was available to this analysis for the following periods of time: the mornings and afternoons of October 13 and October 14, 1971, and the morning of October 19, 1971. Prior to data tabulation, it was decided to estimate roadway costs for the morning and afternoon of October 13.

Moving vehicle study data was available for the period

April 28, 1970 to January 27, 1972; during this period, eleven individuals operated the instrumented vehicle for a total of one hundred and seventy-eight runs. Prior to data tabulation, it was decided to minimize the impact of vehicle operator variability by considering the data associated with the operator who accounted for the greatest number of runs; the operator chosen (Welch) totalled fifty-four runs during the period April 28, 1970 to November 16, 1970.

Data availability, therefore, precluded the synthesis of macroscopic and microscopic data for a unique time period. Since, however, the geometrics of the roadway section remained unaltered during the transpiration of the two periods, one is able to dismiss this problem of noncontiguous time periods.

Data analysis

Both the time cost and operating cost components of the optimal roadway price schedule are estimated by application of the general linear regression model (70). The approach for estimating the time cost component is based upon the assumption that a system of vehicle units will perform in a manner consistent with the fundamental diagram.

The consideration of the fundamental diagram of road traffic in the previous chapter indicates that from a theoretical point of view no one of the relevant variables -- flow, concentration, and speed -- is strictly defined to be dependent or independent with respect to the others. In the real world, however, the transportation cost incurred by the motorist varies with traffic flow. Beckmann, McGuire and Winsten explain this relationship, as follows (3, p. 49):

> This dependence may be described very simply by saying that an individual road user is the worse off, the more traffic there is on the road he is traveling. This is so because the presence of other traffic causes delays, added risks, and extra operating cost as expended in passing maneuvers. The road user incurs a higher transportation cost the larger the flow, even though a lower average speed may actually reduce his operating cost and although risks, as measured in accidents per vehicle mile, may start falling off at a flow level where passing becomes increasingly rare. The explanation is of course that these latter cost reductions, if any, from congestion are more than offset by the accompanying losses of time. By congestion we mean here traffic conditions which occur at flows that "substantially" reduce average speed on the road.

Recalling that the present analysis assumes the roadway to be a fixed-capacity facility, Figure 12 (149, p. 24; 152, p. 679; 161, p. 301) can be used to generally portray the relationship between vehicular system flow and travel cost. The short run average cost and short run marginal cost curves are defined as those in Figure 2. The critical flow, $q_m$, is typically defined as the "capacity" of the roadway: it is the maximum level of flow able to be physically sustained. As indicated by the fundamental diagram, an increase in vehicular concentration beyond that level consistent with critical flow gives rise to a proportionately larger decrease in system speed, and flow falls while unit cost increases;

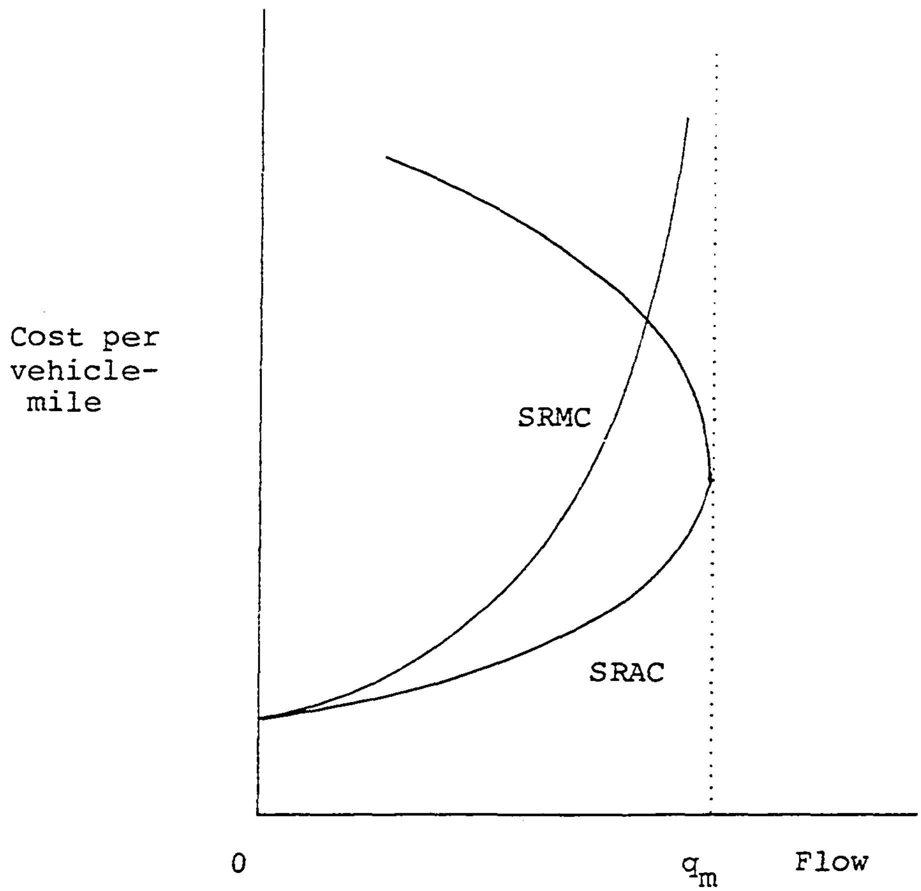


Figure 12. Flow-cost relationship for a fixed-capacity facility

that is, the negatively sloping portion of SRAC is associated with negative changes in output. The SRMC curve is, therefore, not defined for the backward-sloping section of the SRAC curve. When system flow reaches critical flow, SRMC is infinitely high; furthermore, one is able, notes Walters (152, p. 680), to suppose that the short run marginal cost "is infinite for those levels of flow which are associated with a unit cost higher than that at" the level of flow $q_m$. The theory of marginal cost pricing suggests the desirability of imposing roadway price in order to preclude a rate of vehicular flow entering such a fixed-capacity facility great enough to force system flow to approach too closely the roadway capacity. The time cost component is estimated, here, under the assumed dependence of unit transportation cost upon system flow.

In the analysis of the observed concentration-speed data, the linear (that is, with n = 1) roadway model reviewed in the previous chapter is employed. Equation 23

$$u = u_f(1 - \frac{k}{k_j})$$

can be rewritten as

$$u = a_1 + a_2k \qquad 47$$

where:

$$a_1 = u_f;$$

$$a_2 = -\frac{u_f}{k_j} .$$

Table 1 summarizes the results of the concentration-speed regression. The predicted levels of concentration and speed consistent with the maximum level of flow for a given state of vehicular traffic -- that is, the predicted "critical" levels of concentration, speed, and flow -- are calculated by utilizing Equations 26-28,

$$k_m = \frac{1}{2} k_j ,$$

$$u_m = \frac{1}{2} u_f ,$$

and

$$q_m = \frac{1}{4} k_j u_f ,$$

respectively. Figure 13 shows the observed concentration-speed data and the linear regression for October 19.

The observed concentration-speed data is able to be used to generate flow data by appealing to Equation 1,

$$q = ku .$$

Considering the relationship between the flow data computed in this manner and the observed concentration data, Equation 24

$$q = ku_f(1 - \frac{k}{k_j})$$

is specified in the form

$$q = a_3 + a_4 k + a_5 k^2 \qquad 48$$

Table 1. Regression analysis of concentration-speed data

| Regression | Intercept ($a_1$) | Regression Coefficient ($a_2$) | t-value[a] | F-value[a] |
|---|---|---|---|---|
| 10-13-71 A.M. | 66.600 | -0.6218 | -5.577 (-2.756) | 31.099 (7.56) |
| 10-13-71 P.M. | 72.373 | -0.9945 | -5.173 (-2.724) | 26.761 (7.56) |
| 10-19-71 A.M. | 44.355 | -0.2856 | -11.335 (-2.756) | 128.48 (7.56) |

[a]Critical values at the $\alpha = 0.01$ level are indicated in parentheses.

| Multiple Correlation | Predicted Roadway Parameters: Free Speed ($u_f$) | Jam Concentration ($k_j$) | Critical Flow ($q_m$) | Critical Concentration ($k_m$) | Critical Speed ($u_m$) |
|---|---|---|---|---|---|
| 0.7194 | 66.60 | 107.11 | 1783.35 | 53.55 | 33.30 |
| 0.6479 | 72.37 | 72.77 | 1316.70 | 36.39 | 36.19 |
| 0.9032 | 44.36 | 155.31 | 1722.14 | 77.65 | 22.18 |

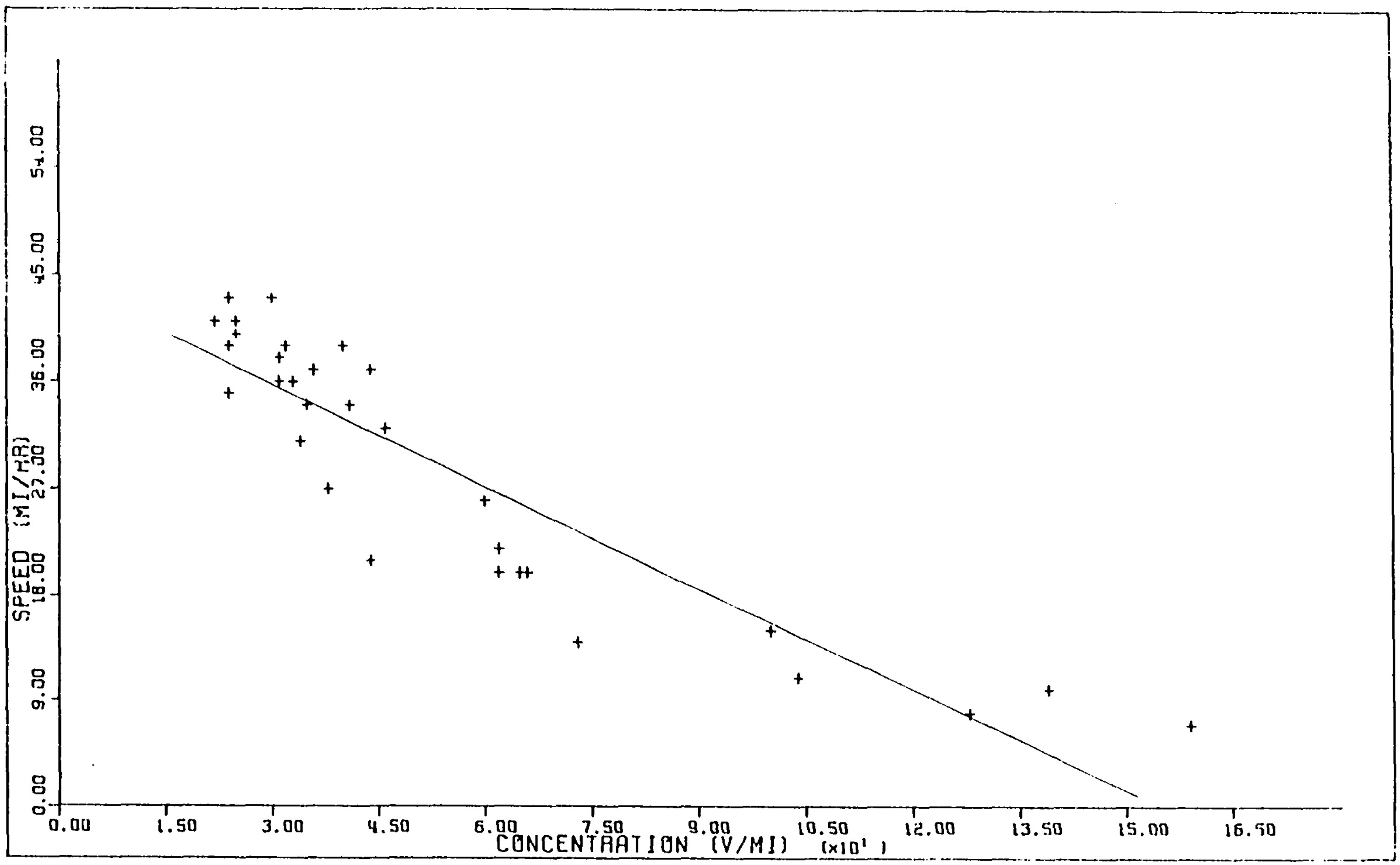


Figure 13. Concentration-speed relationship for 10-19-71 A.M.

where:

$$a_3 = 0;$$

$$a_4 = u_f;$$

$$a_5 = -\frac{u_f}{k_j} .$$

Table 2 summarizes the results of the concentration-flow regression; the predicted critical roadway parameters are computed by applying Equations 26-28. Figure 14 shows the observed concentration-flow data and the linear regression for October 19.

Recalling that time is the reciprocal of speed, Equation 44

$$t = t(k)$$

is able to be specified by employing Equation 47; the expression of time in terms of minutes per mile results in

$$t^* = a_6 + a_7 k \qquad 49$$

where:

$$t^* = \frac{1}{t};$$

$$a_6 = \frac{a_1}{60};$$

$$a_7 = \frac{a_2}{60} .$$

Table 2. Regression analysis of concentration-flow data

| Regression | Intercept ($a_3$) | Regression Coefficient ($a_4$) | ($a_5$) | t-value[a] | F-value[a] |
|---|---|---|---|---|---|
| | 0.0 | | | | |
| | | 67.8120 | | 21.376 (2.756) | |
| 10-13-71 A.M. | | | -0.6545 | -8.309 (-2.756) | 1320.2 (5.39) |
| | 0.0 | | | | |
| | | 67.0444 | | 15.770 (2.724) | |
| 10-13-71 P.M. | | | -0.7603 | -4.332 (-2.724) | 1463.7 (5.39) |
| | 0.0 | | | | |
| | | 37.5454 | | 18.912 (2.756) | |
| 10-19-71 A.M. | | | -0.2098 | -11.869 (-2.756) | 276.65 (5.39) |

[a]Critical values at the $\alpha = 0.01$ level are indicated in parentheses.

| Multiple Correlation | Predicted Roadway Parameters: Free Speed ($u_f$) | Jam Concentration ($k_j$) | Critical Flow ($q_m$) | Critical Concentration ($k_m$) | Critical Speed ($u_m$) |
|---|---|---|---|---|---|
| 0.9946 | 67.81 | 103.61 | 1756.48 | 51.81 | 33.91 |
| 0.9937 | 67.04 | 88.18 | 1478.02 | 44.09 | 33.52 |
| 0.9748 | 37.55 | 178.96 | 1679.76 | 89.48 | 18.77 |

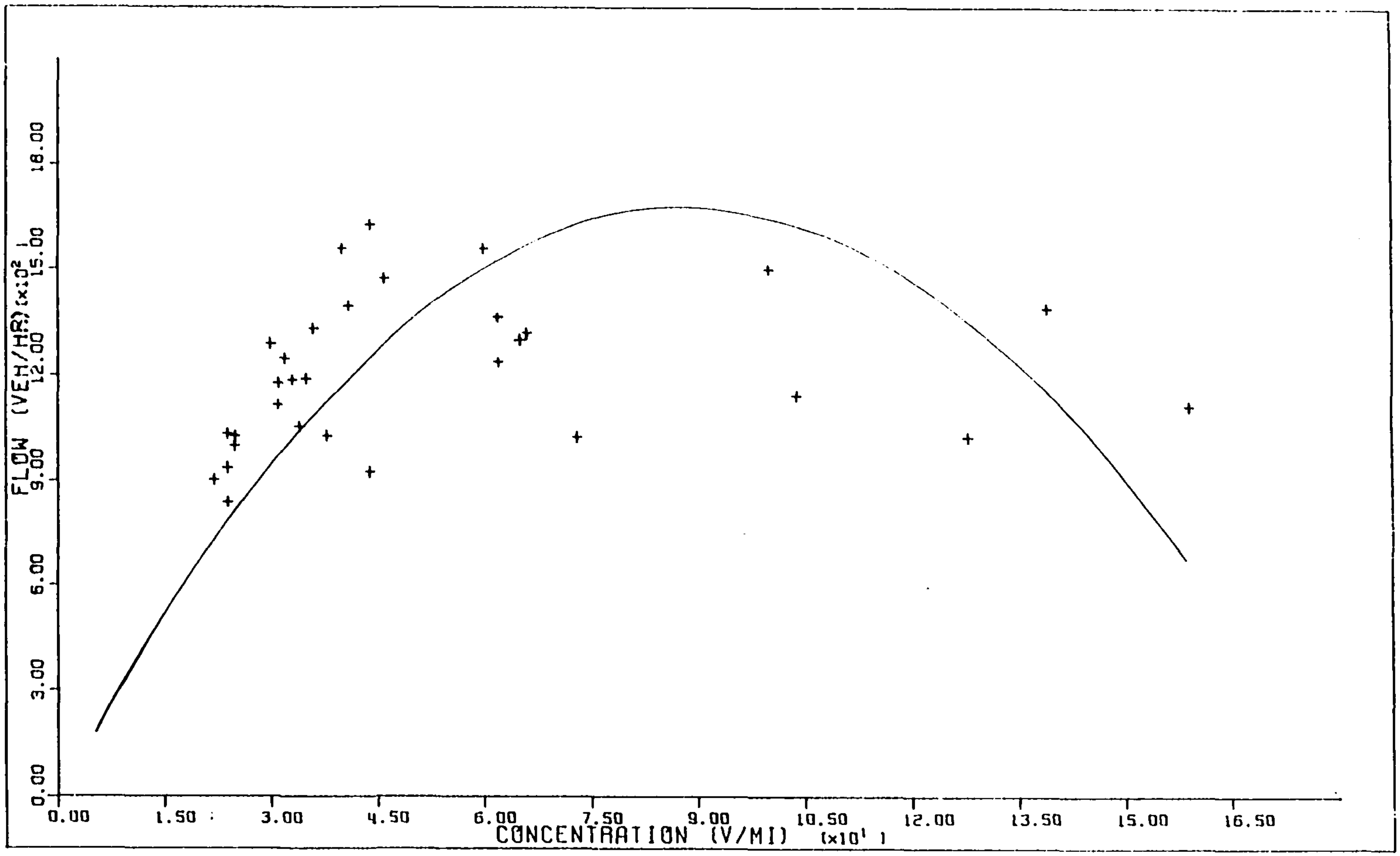


Figure 14. Concentration-flow relationship for 10-19-71 A.M.

Table 3 summarizes the results of the concentration-time regression; since t* is an arithmetic transformation of u, the regression analyses of Equations 47 and 49 are identical. Figure 15 shows the observed concentration-time data and the linear regression for October 19. The results of this regression are to be utilized in the generation of the time cost component of the optimal roadway price schedule.

As in the case of the time cost component, the operating cost function is estimated, here, under the assumed dependence of unit transportation cost upon system flow. From the definition of acceleration noise given as Equation 8

$$\sigma = \{\frac{1}{T} \int_0^T [a(t_i)-a_{ave}]^2 dt\}^{1/2}$$

it follows that

$$\sigma^2 = \frac{1}{T} \int_0^T [a(t_i)-a_{ave}]^2 dt;$$

expanding and simplifying,

$$\sigma^2 = \frac{1}{T} \int_0^T [a(t_i)]^2 dt - [a_{ave}]^2 .$$

This result can be approximated by using

$$\sigma^2 = \frac{1}{T} \sum_{i=0}^{T} [\frac{\Delta u(t_i)}{\Delta t_i}]^2 \Delta t_i - [\frac{u(T)-u(0)}{T}]^2;$$

if, throughout the measurement, $\Delta u$ is specified as a constant, then

Table 3. Regression analysis of concentration-time data

| Regression | Intercept ($a_6$) | Regression Coefficient ($a_7$) | t-value[a] | F-value[a] | Multiple Correlation |
|---|---|---|---|---|---|
| 10-13-71 A.M. | 1.110 | -0.0104 | -5.577<br>(-2.756) | 31.099<br>(7.56) | 0.7194 |
| 10-13-71 P.M. | 1.206 | -0.0166 | -5.173<br>(-2.724) | 26.761<br>(7.56) | 0.6479 |
| 10-19-71 A.M. | 0.739 | -0.0048 | -11.335<br>(-2.756) | 128.48<br>(7.56) | 0.9032 |

[a]Critical values at the $\alpha = 0.01$ level are indicated in parentheses.

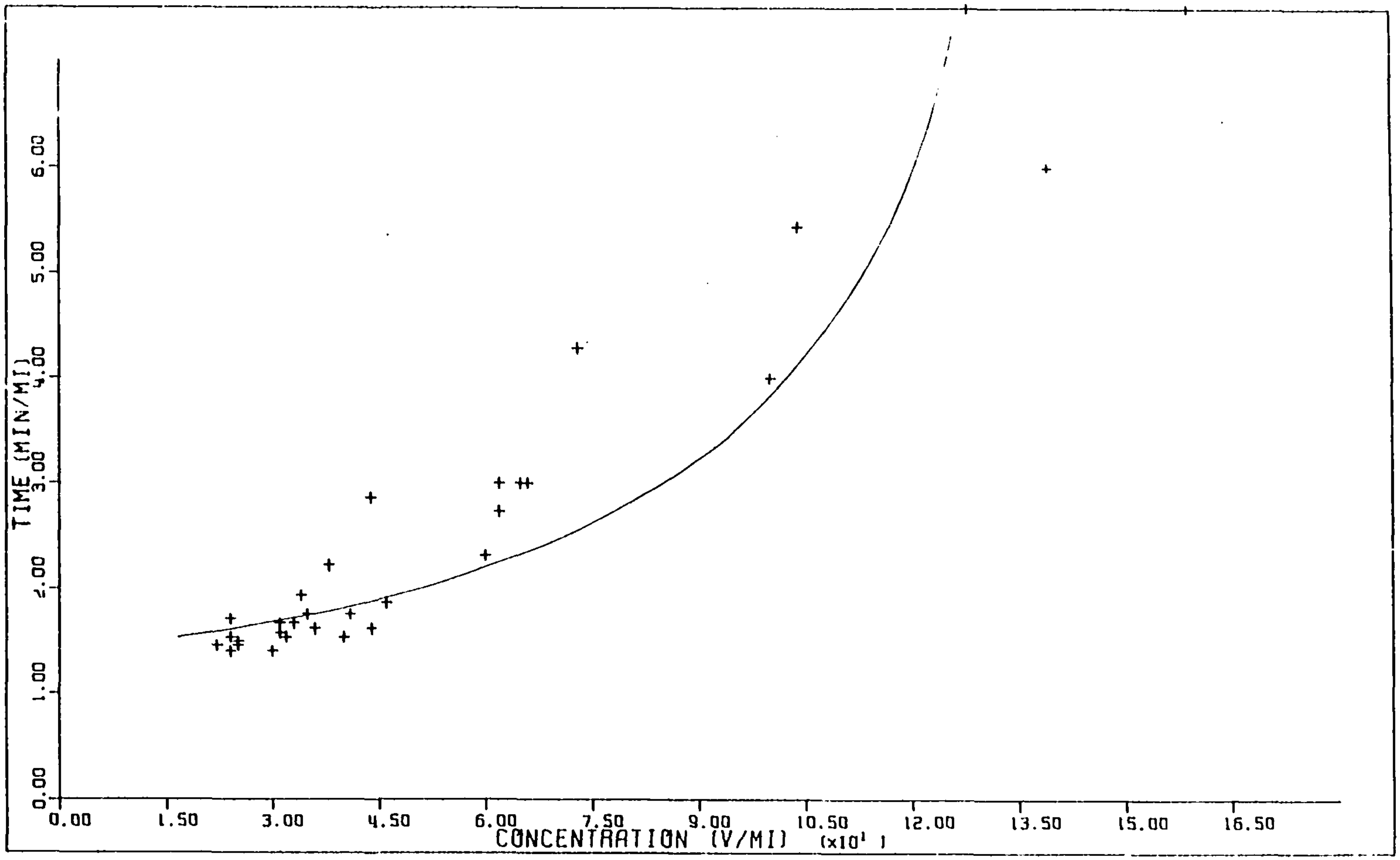


Table 15. Concentration-time relationship for 10-19-71 A.M.

$$\sigma^2 = \frac{(\Delta u)^2}{T} \sum_{i=0}^{T} \frac{n^2}{\Delta t_i} - [\frac{u(T)-u(0)}{T}]^2,$$

where n denotes the number of speed changes of $\Delta u$ occurring in $\Delta t_i$; if $\Delta t$ is measured for each speed change of $\Delta u$, then

$$\sigma^2 = \frac{(\Delta u)^2}{T} \sum_{i=0}^{T} \frac{1}{\Delta t_i} - [\frac{u(T)-u(0)}{T}]^2;$$

and if, finally, the roadway section under consideration is very long (that is, for T having a large magnitude), or if the final speed equals the initial speed, then (29, 72) the Potts-Jones approximation of Equation 8 becomes

$$\sigma = \{\frac{(\Delta u)^2}{T} \sum_{i=0}^{T} \frac{1}{\Delta t_i}\}^{1/2} \quad .$$

This approximation takes T as the total running time of the trip (14), whereas Equation 8 interprets T to be the total trip time.

For short sections of freeway, it is unrealistic to expect the final and initial speeds to be equal. However, it has been shown (31) that if acceleration noise is measured on short sections with respect to zero acceleration, rather than with respect to $a_{ave}$, the previous result applies to this case as well.

Moreover, if it is measured with respect to zero acceleration, acceleration noise exhibits the following additive property: in general, if $\sigma$ has been calculated for N successive roadway subsystems, then $\sigma$ for the roadway section which consists of these subsystems is computed as

$$\sigma = \left\{ \frac{(\Delta u)^2 \sum_{j=1}^{N} \sum_{i=0}^{T} \frac{1}{\Delta t_i}}{\sum_{j=1}^{N} T_j} \right\}^{1/2}$$

where:

$j = 1,\ldots,N$ successive subsystems;

$T_j$ = the total running time of the trip over subsystem $j$.

The data available, however, specified neither $\Delta u$ nor $\Delta t_i$ for subsystems; the previous result, therefore, is herein modified and employed in the form

$$\sigma = \left\{ \frac{\sum_{j=1}^{N} T_j \sigma_j^2}{\sum_{j=1}^{N} T_j} \right\}^{1/2}$$

where:

$\sigma_j$ = the acceleration noise of subsystem $j$.

The application of this result allowed aggregation of the data related to the two individual and contiguous microscopic subsystems to which this study had access.

In the analysis of the observed speed-acceleration noise and speed-fuel consumption data, the linear (that is, with n=1) model reviewed in the previous chapter is employed. With respect to the former, Equation 38

$$\sigma = \sigma_{max} - \frac{27}{4}\ \sigma_{max}[(\frac{u}{u_f})^2 - (\frac{u}{u_f})^3]$$

is specified in the form

$$\sigma = b_1 + b_2 u + b_3 u^2 + b_4 u^3 \qquad 50$$

where:

$b_1 = \sigma_{max}$;

$b_2 = 0$;

$b_3 = -\frac{27}{4}\ \frac{\sigma_{max}}{u_f^2}$;

$b_4 = \frac{27}{4}\ \frac{\sigma_{max}}{u_f^3}$.

Table 4 summarizes the results of the speed-acceleration noise regression, as well as samples of regression results of Capelle (14, p. 94) and Drew, Dudek, and Keese (31, p. 61). The computation of the predicted level of speed consistent with minimum acceleration noise -- that is, the predicted "optimum" speed -- is able to be executed by solving for that level of speed which equates the first derivative of Equation 50 with zero. The computation of the predicted free speed utilizes Equation 36,

$$u'_m = \frac{2}{3}\,u_f\ .$$

Table 4. Regression analysis of speed-acceleration noise data

| Regression | Intercept ($b_1$) | Regression Coefficients ($b_3 x10^2$) | ($b_4 x10^4$) | t-value | F-value |
|---|---|---|---|---|---|
| | 2.1359 | | | | |
| | | -0.1161 | | -2.235 (-2.008)[a] | |
| Equation 50 | | | 0.1656 | 1.998 (1.676)[b] | 4.526 (4.31)[c] |
| | 2.32 | | | | |
| | | -0.31 | | -12.10[d] | |
| Capelle[c] | | | 0.46 | 10.14[d] | 120[d] |
| | 1.67 | | | | |
| | | -0.25 | | -4.99[d] | |
| Capelle[c] | | | 0.36 | 4.74[d] | 13[d] |
| Drew, Dudek and Keese | 1.289 | -0.657 | 0.419 | | |
| Drew Dudek and Keese | 2.314 | -0.3159 | 0.4607 | -[d] -[d] | -[d] |
| Drew Dudek and Keese | 1.949 | -0.4358 | 0.8022 | -[d] -[d] | -[d] |

[a]Critical value at the α=0.05 level.

[b]Critical value at the α=0.10 level.

[c]Critical values at the α=0.01 level are indicated in parentheses.

[d]Indicates significance at the α=0.01 level.

| Multiple Correlation | Predicted Roadway Parameters: Maximum Acceleration Noise ($\sigma_{max}$) | Optimum Speed ($u_m'$) | Free Speed ($u_f$) |
|---|---|---|---|
| 0.3883 | 2.1359 | 46.739 | 70.109 |
| 0.79 | 2.32 | 44.927 | 67.391 |
| 0.38 | 1.67 | 46.296 | 69.444 |
| 0.61 | 1.289 | 104.532 | 156.798 |
| 0.38 | 2.314 | 45.713 | 68.570 |
| 0.60 | 1.949 | 36.217 | 54.326 |

Figure 16 exhibits the observed speed-acceleration noise data and the linear regression.

With respect to the speed-fuel consumption data, Equation 45

$$O.C. = O.C._{max} - \frac{27}{4}\ O.C._{max}[(\frac{u}{u_f})^2 - (\frac{u}{u_f})^3]$$

is able to be written in the form

$$O.C. = b_5 + b_6u + b_7u^2 + b_8u^3 \qquad 51$$

where:

$b_5 = O.C._{max}$;

$b_6 = 0$;

$b_7 = -\frac{27}{4}\ \frac{O.C._{max}}{u_f^2}$;

$b_8 = \frac{27}{4}\ \frac{O.C._{max}}{u_f^3}$ .

Considering only the fuel component of the total vehicular operating cost, the results of the speed-fuel consumption regression are tabulated in Table 5; the regression results of Capelle (14, p. 97) are also shown. The predicted optimum and free speeds are calculated as in Table 4. Figure 17 presents the observed speed-fuel consumption data and linear regression. The results of this regression are to be

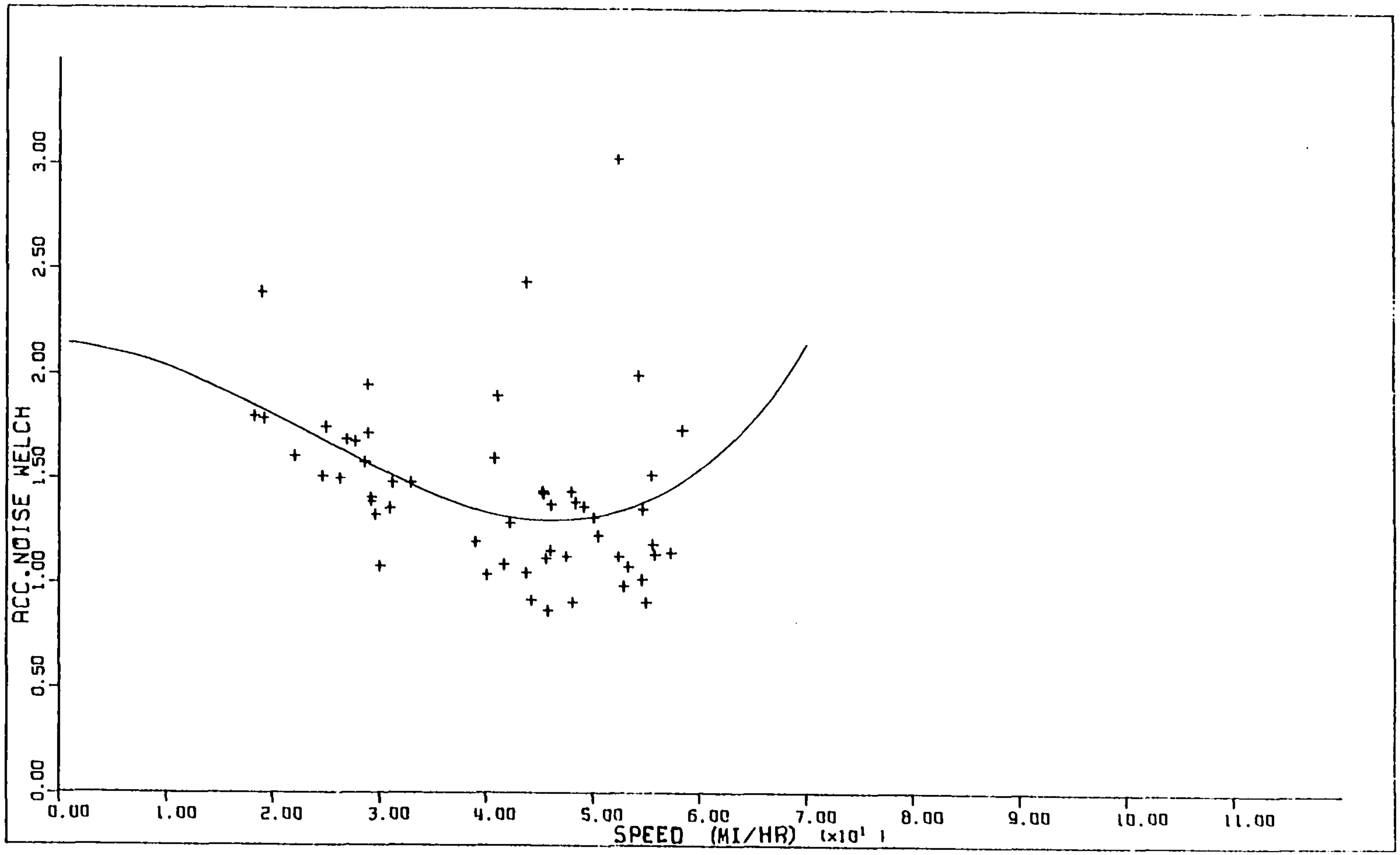


Figure 16. Speed-acceleration noise relationship

Table 5. Regression analysis of speed-fuel consumption data

| Regression | Intercept ($b_5 x10$) | Regression Coefficients ($b_7 x10^4$) | ($b_8 x10^6$) | t-value[a] | F-value[a] |
|---|---|---|---|---|---|
| | 0.8125 | | | | |
| | | -0.5859 | | -5.674 (-2.678) | |
| Equation 51 | | | 0.7593 | 4.607 (2.678) | 59.240 (4.31) |
| | 10.7 | | | | |
| | | -6 | | -14.67[b] | |
| Capelle | | | 9 | 14.13[b] | 114.8[b] |

[a]Critical values at the $\alpha$=0.01 level are indicated in parentheses.

[b]Indicates significance at the $\alpha$=0.01 level.

| Multiple Correlation | Predicted Roadway Parameters | | |
|---|---|---|---|
| | $O.C._{max}$ | Optimum Speed ($u'_m$) | Free Speed ($u_f$) |
| 0.8361 | 0.8125 | 51.4419 | 77.163 |
| 0.81 | 1.07 | 44.444 | 66.667 |

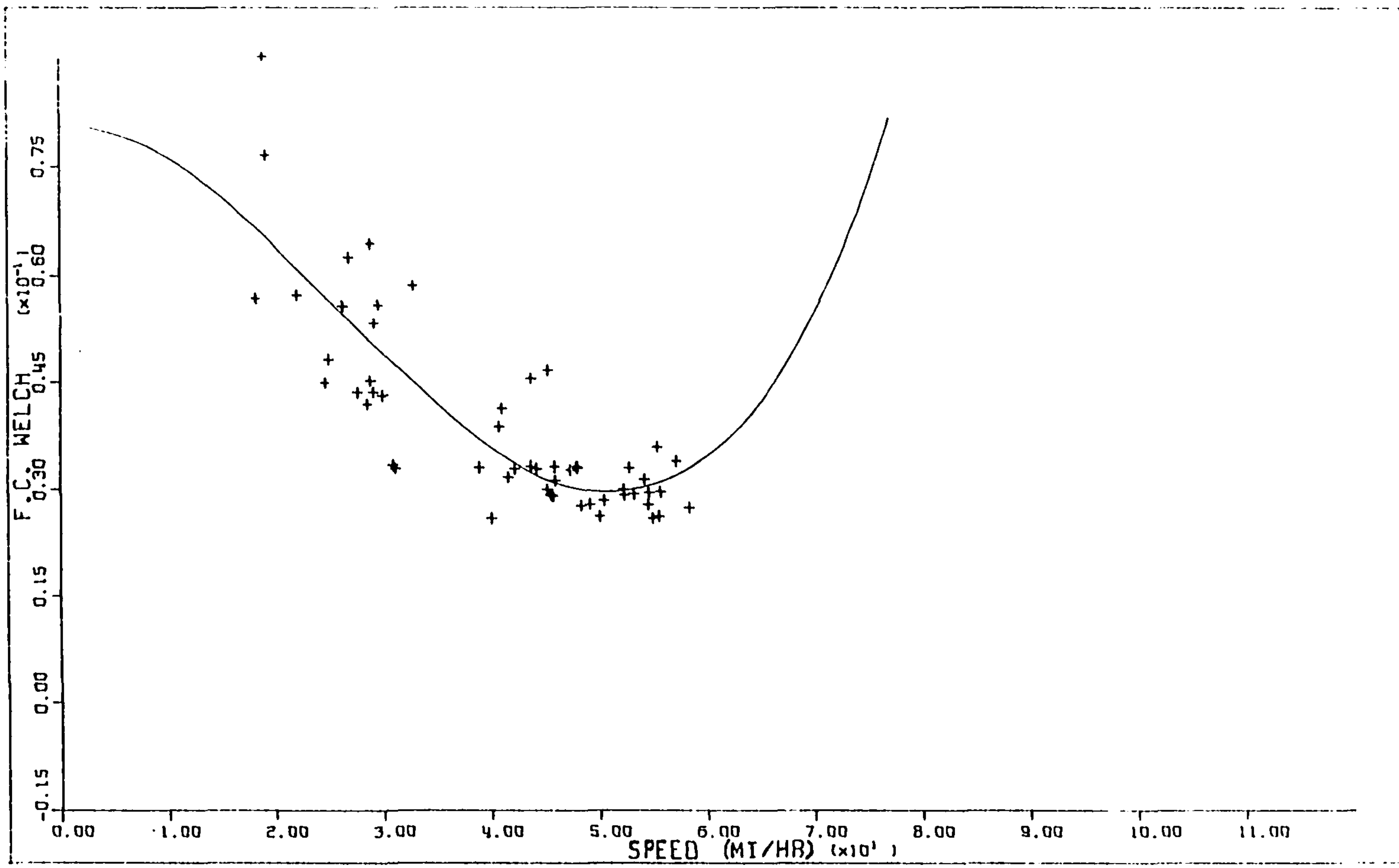


Figure 17. Speed-fuel consumption relationship

utilized in the generation of the operating cost component of the optimal roadway price schedule.

To test the consistency of the macroscopic and the microscopic data for the roadway section under consideration, one is able to compare the predicted critical and optimum speeds by applying Equation 40,

$$u'_m = \frac{4}{3} u_m .$$

This comparison is exhibited in Table 6.

## Estimation of the Roadway Cost and Toll Schedules

Having estimated the average social time cost and the average social operating cost functions, one is in a position to generate the relevant marginal cost functions and tolls, by the application of Equation 46,

$$MSTC = ASTC\ (1 + \varepsilon_{ASTC})$$

and by the application of the analogous result,

$$MSOC = ASOC\ (1 + \varepsilon_{ASOC}) \qquad 52$$

where:

$\varepsilon_{ASOC}$ = elasticity of average social operating cost.

Tables 7 and 8 present the schedule of the time cost component of the optimal toll associated with levels of vehicular system flow and speed, as generated by the concentration-flow and

Table 6. Comparison of predicted critical and optimum speeds

| Regression | Critical Speed | Optimum Speed |
|---|---|---|
| Macroscopic data | | |
| Concentration-speed | | |
| 10-13-71 A.M. | 33.30 | 44.40 |
| 10-13-71 P.M. | 36.19 | 48.25 |
| Concentration-flow | | |
| 10-13-71 A.M. | 33.91 | 45.21 |
| 10-13-71 P.M. | 33.52 | 44.69 |
| Microscopic data | | |
| Speed-acceleration noise | 35.05 | 46.74 |
| Speed-fuel consumption | 38.58 | 51.44 |

concentration-time regressions: ASTC is the linear regression of concentration and time. The point elasticity of the average social time cost function is calculated from the regression analyses of Equations 48 and 49 as follows: from Equation 48,

$$\varepsilon_{qk} = \frac{dq}{q} \Big/ \frac{dk}{k}$$

$$= k\left[\frac{dq}{dk}\right] \frac{1}{q}$$

$$= k\left[\frac{d}{dk}(a_4k+a_5k^2)\right] \frac{1}{a_4k+a_5k^2}$$

$$= \frac{a_4k+2a_5k^2}{a_4k+a_5k^2} ;$$

and from Equation 49,

Table 7. Estimated time cost schedules for 10-13-71 A.M.

| System Flow (vehicles/ hour) | System Speed (miles/ hour) | ASTC (minutes/ mile) | $\varepsilon_{ASTC}$ | MSTC (minutes/ mile) | Time Cost Component of Optimal Toll (minutes/ mile) | Time Cost Component of Optimal Toll (cents/ mile) |
|---|---|---|---|---|---|---|
| 195 | 65 | 0.927 | 0.031 | 0.956 | 0.029 | 0.076 |
| 660 | 60 | 1.004 | 0.161 | 1.165 | 0.161 | 0.418 |
| 1045 | 55 | 1.095 | 0.403 | 1.536 | 0.441 | 1.143 |
| 1350 | 50 | 1.204 | 0.887 | 2.273 | 1.069 | 2.768 |
| 1575 | 45 | 1.338 | 2.002 | 4.016 | 2.678 | 6.937 |
| 1720 | 40 | 1.505 | 5.584 | 9.908 | 8.403 | 21.764 |
| 1785 | 35 | 1.719 | 87.254 | 151.734 | 150.014 | 388.537 |
| 1770 | 30 | 2.005 | ∞ | ∞ | ∞ | ∞ |

Table 8. Estimated time cost schedules for 10-13-71 P.M.

| System Flow (vehicles/ hour) | System Speed (miles/ hour) | ASTC (minutes/ mile) | $\varepsilon_{ASTC}$ | MSTC (minutes/ mile) | Time Cost Component of Optimal Toll (minutes/ mile) | Time Cost Component of Optimal Toll (cents/ mile) |
|---|---|---|---|---|---|---|
| 455 | 65 | 0.917 | 0.137 | 1.043 | 0.125 | 0.324 |
| 720 | 60 | 0.993 | 0.308 | 1.299 | 0.306 | 0.793 |
| 935 | 55 | 1.082 | 0.592 | 1.722 | 0.640 | 1.658 |
| 1100 | 50 | 1.188 | 1.081 | 2.473 | 1.284 | 3.326 |
| 1260 | 45 | 1.348 | 2.258 | 4.391 | 3.043 | 7.881 |
| 1320 | 40 | 1.517 | 4.533 | 8.395 | 6.878 | 17.813 |
| 1330 | 35 | 1.736 | 11.320 | 21.382 | 19.647 | 50.885 |
| 1290 | 30 | 2.027 | ∞ | ∞ | ∞ | ∞ |

$$\varepsilon_{tk} = \frac{dt}{t} \Big/ \frac{dk}{k}$$

$$= k\left[\frac{dt}{dk}\right] \frac{1}{t}$$

$$= k\left[\frac{d}{dk}\left(\frac{60}{a_1+a_2k}\right)\right] \frac{a_1+a_2k}{60}$$

$$= \frac{-a_2k}{a_1+a_2k} \quad .$$

With the application of the chain rule of elasticity and of "the theorem which states that the elasticity of a function with respect to its independent variable is equal to the reciprocal of the elasticity of the inverse function with respect to its independent variable" (68, p. 146), it follows that

$$\varepsilon_{ASTC} = \varepsilon_{tq}$$

$$= \varepsilon_{tk} \cdot \varepsilon_{kq}$$

$$= \varepsilon_{tk} \; \frac{1}{\varepsilon_{qk}}$$

$$= \left(\frac{-a_2k}{a_1+a_2k}\right)\left(\frac{a_4k+a_5k^2}{a_4k+2a_5k^2}\right)$$

$$= - \frac{a_2a_4k+a_2a_5k^2}{a_1a_4+(2a_1a_5+a_2a_4)k + 2a_2a_5k^2} \quad .$$

The application of Equation 46 allows computation of MSTC and the time cost component, both expressed in terms of minutes per mile; the time cost component is a monotonically increasing function of flow, and a monotonically decreasing function of speed (9, 26).

The literature related to the problem of measuring the value of commuting time is replete (2, 5, 49, 69, 98, 100, 128, 135). The present analysis uses the value of time established by AASHO, the American Association of State Highway Officials (23, p. 126):

> The dollar value of time saving may vary considerably and no precise method of evaluation yet has been determined. A value of time for passenger cars of $1.55 per hour, or 2.59 cents per minute, is used herein as representative of current opinion for a logical and practical value. The typical passenger car has 1.8 persons in it, and a time value of $0.86 per person per hour results in a vehicle total of $1.55 per hour.

Accepting this relatively conservative (8, 12, 20, 23, 94, 133, 141, 157, 161) estimate of 2.59 cents per vehicle minute, the optimal time cost component is tranformed from a minutes-per-mile to a cents-per-mile dimension.

Tables 9 and 10 display the schedule of the operating cost component of the optimal toll associated with levels of vehicular system flow and speed, as generated by the concentration-speed and speed-fuel consumption regressions: ASFC is the linear regression of speed and fuel consumption. The point elasticity of the average social fuel consumption function is computed from the regression analyses of Equations 47 and 51 as follows: from Equations 1 and 47,

Table 9. Estimated operating cost schedules for 10-13-71 A.M.

| System Flow (vehicles/hour) | System Speed (miles/hour) | ASFC (gal./mile) | $\varepsilon_{ASFC}$ | MSFC (gallons/mile) | Fuel Cost Component of Optimal Toll (gallons/mile) | Fuel Cost Component of Optimal Toll (cents/mile) | Operating Cost Component of Optimal Toll (cents/mile) |
|---|---|---|---|---|---|---|---|
| 195 | 65 | 0.042 | -0.078 | 0.039 | -0.003 | -0.105 | -0.210 |
| 660 | 60 | 0.034 | -0.253 | 0.026 | -0.009 | -0.278 | -0.556 |
| 1045 | 55 | 0.030 | -0.216 | 0.024 | -0.007 | -0.210 | -0.420 |
| 1350 | 50 | 0.030 | 0.137 | 0.034 | 0.004 | 0.130 | 0.260 |
| 1575 | 45 | 0.031 | 0.862 | 0.059 | 0.027 | 0.877 | 1.754 |
| 1720 | 40 | 0.036 | 2.292 | 0.119 | 0.083 | 2.649 | 5.298 |
| 1785 | 35 | 0.042 | 10.142 | 0.468 | 0.426 | 13.644 | 27.288 |
| 1770 | 30 | 0.049 | ∞ | ∞ | ∞ | ∞ | ∞ |

Table 10. Estimated operating cost schedules for 10-13-71 P.M.

| System Flow (vehicles/hour) | System Speed (miles/hour) | ASFC (gal./mile) | $\varepsilon_{ASFC}$ | MSFC (gallons/mile) | Fuel Cost Component of Optimal Toll (gallons/mile) | Fuel Cost Component of Optimal Toll (cents/mile) | Operating Cost Component of Optimal Toll (cents/mile) |
|---|---|---|---|---|---|---|---|
| 455 | 65 | 0.042 | -0.395 | 0.026 | -0.017 | -0.534 | -1.068 |
| 720 | 60 | 0.034 | -0.531 | 0.016 | -0.018 | -0.584 | -1.168 |
| 935 | 55 | 0.030 | -0.373 | 0.019 | -0.011 | -0.362 | -0.724 |
| 1100 | 50 | 0.030 | 0.244 | 0.036 | 0.007 | 0.213 | 0.426 |
| 1260 | 45 | 0.031 | 1.450 | 0.078 | 0.046 | 1.476 | 2.952 |
| 1320 | 40 | 0.036 | 4.902 | 0.213 | 0.177 | 5.664 | 11.328 |
| 1330 | 35 | 0.042 | ∞ | ∞ | ∞ | ∞ | ∞ |
| 1290 | 30 | 0.049 | | | | | |

$$u = a_1 + a_2 k$$

$$= a_1 + a_2 \frac{q}{u}$$

or, in terms of flow,

$$q = -\frac{a_1}{a_2} u + \frac{1}{a_2} u^2; \qquad 53$$

therefore,

$$\varepsilon_{qu} = \frac{dq}{q} \Big/ \frac{du}{u}$$

$$= u[\frac{dq}{du}] \frac{1}{q}$$

$$= u[\frac{d}{du}(-\frac{a_1}{a_2} u + \frac{1}{a_2} u^2)] \quad \frac{1}{-\frac{a_1}{a_2} u + \frac{1}{a_2} u^2}$$

$$= \frac{-\frac{a_1}{a_2} u + \frac{2}{a_2} u^2}{-\frac{a_1}{a_2} u + \frac{1}{a_2} u^2} \quad ;$$

and from Equation 51,

$$\varepsilon_{F.C.,u} = \frac{dF.C.}{F.C.} \Big/ \frac{du}{u}$$

$$= u[\frac{dF.C.}{du}] \frac{1}{F.C.}$$

$$= u[\frac{d}{du}(b_5+b_7u^2+b_8u^3)] \frac{1}{b_5+b_7u^2+b_8u^3}$$

$$= \frac{2b_7u^2+3b_8u^3}{b_5+b_7u^2+b_8u^3} \ .$$

It follows that

$$\varepsilon_{ASFC} = \varepsilon_{F.C.,q}$$

$$= \varepsilon_{F.C.,u} \cdot \varepsilon_{uq}$$

$$= \varepsilon_{F.C.,u} \frac{1}{\varepsilon_{qu}}$$

$$= \left(\frac{2b_7u^2+3b_8u^3}{b_5+b_7u^2+b_8u^3}\right)\left(\frac{-\frac{a_1}{a_2}u + \frac{1}{a_2}u^2}{-\frac{a_1}{a_2}u + \frac{2}{a_2}u^2}\right)$$

$$= \frac{\alpha_0u^3+\alpha_1u^4+\alpha_2u^5}{\alpha_3u+\alpha_4u^2+\alpha_5u^3+\alpha_6u^4+\alpha_7u^5}$$

where:

$$\alpha_0 = -2\frac{a_1}{a_2}b_7;$$

$$\alpha_1 = 2\frac{1}{a_2}b_7 - 3\frac{a_1}{a_2}b_8;$$

$$\alpha_2 = 3\frac{1}{a_2}b_8;$$

$$\alpha_3 = -\frac{a_1}{a_2}b_5;$$

$$\alpha_4 = 2\frac{1}{a_2}b_5;$$

$$\alpha_5 = - \frac{a_1}{a_2} b_7;$$

$$\alpha_6 = 2 \frac{1}{a_2} b_7 - \frac{a_1}{a_2} b_8;$$

$$\alpha_7 = 2 \frac{1}{a_2} b_8 .$$

The application of Equation 52 allows computation of MSFC and the fuel cost component of the optimal toll, both expressed in terms of gallons per mile.

The relationship between average fuel consumption cost (14, p. 110), AFC, and the relevant marginal function, MFC, is displayed in Figure 18. The speed-acceleration noise model is shown in Figure 7 above. Fuel consumption has been seen to exhibit the same relationship with speed as does acceleration noise. That is, for example, vehicles incur relatively high fuel cost at $u_f$ (under conditions of free flow), enjoy decreasing fuel cost as speed falls from $u_f$ to $u_m'$ (under conditions of increasing system flow), incur increasing fuel cost as speed falls from $u_m'$ (under conditions of increasing system flow until $q_m$ is attained), and incur further increasing fuel cost as speed falls from $u_m$ (under conditions of decreasing system flow) and the motorist is subjected to a situation of stop-and-go. Thus, as seen in Figure 18, each level of flow, aside from $q_m$, has two possible values of fuel cost associated with it. The positively sloped portion

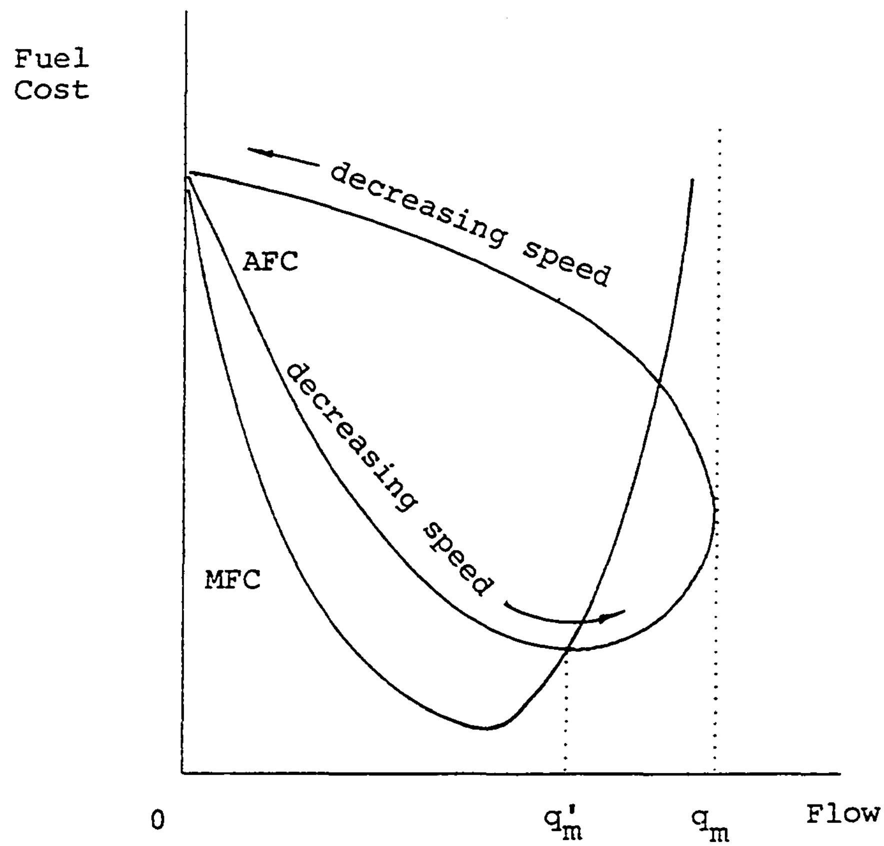


Figure 18. Average fuel consumption and marginal fuel consumption cost relationship

of the fundamental diagram of road traffic (Figure 3) shows speed falling from $u_f$ to $u_m$ as flow increases from zero to $q_m$. Thus in the absence of a posted maximum speed, vehicle units are able to operate at free speed when system flow is low: vehicle units incur a high rate of fuel consumption. Increasing system flow causes speed to fall: the rate of fuel consumption falls, until $u_m'$ is encountered at flow $q_m'$, and thereafter increases. Speed decreases generated by flow increasing within the range $q_m'$ to $q_m$ introduce increasing fuel cost. The section of AFC characterized by decreasing speed with decreasing flow is analogous to the downward-sloping portion of the fundamental diagram and, therefore, is ignored in the analysis as is that portion of the fundamental diagram. The MFC function, then, lies below AFC for flow greater than zero and less than $q_m'$, equals AFC at optimum flow (that is, at the minimum average fuel cost), and lies above AFC for flow greater than $q_m'$.

The present analysis uses the value of vehicle fuel established by AASHO (23, p. 112), which estimates "a gasoline cost of 32 cents per gallon as being reasonably representative for the whole of the United States". The application of this estimate (to the extent that revenue derived from the fuel tax is allocated to support highway facility construction and maintenance expenses, the fuel tax should properly be excluded from this relatively conservative estimate) transforms the optimal operating cost component from a gallons-per-mile to a cents-per-mile dimension. Given that the array of other vehicle operating costs embraces the same relationship with acceleration noise as does fuel consumption, one is in a position to transform this toll schedule to one which accounts for all of these operating costs. It is noted by AASHO (23, p. 75) that in general "the fuel costs approximate 40 to 50 percent of the total vehicle operating cost...and the nonfuel costs are 50 to 60 percent". The present analysis assumes that the fuel cost is 50 percent of total vehicular operating cost.

The total estimated optimal roadway toll schedule for speed increments of 5 miles per hour and associated levels of flow is shown in Table 11. The nonmonotonic nature of the operating cost component causes the schedule of tolls to be nonmonotonic: during the morning (afternoon) period, the optimal price is an increasing function of system flow for levels of speed less than 62 miles per hour (65 miles per hour), and is a decreasing function of flow for levels of speed greater than 62 miles per hour (65 miles per hour).

## Consideration of the Estimated Cost and Toll Schedules

The literature contains a host of time cost and operating cost studies conducted by traffic engineers (10, 59, 63, 99, 115, 126) and economists (27, 39, 47, 94, 116, 152, 157), and

Table 11. Estimated toll schedules for 10-13-71

| System Speed (miles/ hour) | A.M. System Flow (vehicles/ hour) | A.M. Time Cost Component (cents/ mile) | A.M. Operating Cost Component (cents/ mile) | A.M. Optimal Toll (cents/ mile) | P.M. System Flow (vehicles/ hour) | P.M. Time Cost Component (cents/ mile) | P.M. Operating Cost Component (cents/ mile) | P.M. Optimal Toll (cents/ mile) |
|---|---|---|---|---|---|---|---|---|
| 65 | 195 | 0.076 | -0.210 | -0.134 | 455 | 0.324 | -1.068 | -0.726 |
| 60 | 660 | 0.418 | -0.556 | -0.138 | 720 | 0.793 | -1.168 | -0.375 |
| 55 | 1045 | 1.143 | -0.420 | 0.723 | 935 | 1.658 | -0.724 | 0.934 |
| 50 | 1350 | 2.768 | 0.260 | 3.028 | 1100 | 3.326 | 0.426 | 3.752 |
| 45 | 1575 | 6.937 | 1.754 | 8.691 | 1260 | 7.881 | 2.952 | 10.833 |
| 40 | 1720 | 21.764 | 5.298 | 27.062 | 1320 | 17.813 | 11.328 | 29.141 |
| 35 | 1785 | 388.537 | 27.288 | 415.825 | 1330 | 50.885 | ∞ | ∞ |
| 30 | 1770 | ∞ | ∞ | ∞ | 1290 | ∞ | | |

a number of studies (56, 68, 87, 93, 123, 150) related to the estimation of optimal toll schedules. Table 12 compares the cost and toll schedules estimated by the present analysis with those estimated by the following: Haikalis and Joseph (47), Oppenlander (99), Winfrey (157), Johnson (68), and Walters (152).

The average social time cost schedules and time cost component schedules cited in Table 12 are monotonically decreasing functions of speed. The average social operating cost schedules cited are monotonically decreasing functions of speed for levels of speed less than an optimal speed, and are monotonically increasing functions for levels of speed greater than that optimal speed; however, whereas the Haikalis and Joseph, Oppenlander, and Winfrey schedules are based upon historical data and the traditional parabolic speed-operating cost model, the schedule estimated in the present approach is based upon real-time data and the speed-acceleration noise model. Moreover, the present analysis diverges from the customary approach with respect to the degree of motivation provided the traffic system by the operating cost component.

As a rule, an estimated operating cost component schedule discourages vehicle units from using a roadway section characterized by "congestion" -- that is, a roadway section whereon vehicular flow is less than critical flow and vehicular speed is less than critical speed -- by virtue of

Table 12a. A comparison among estimated cost and toll schedules[a]

| System Speed (miles/ hour) | ASTC | | | ASOC | | | |
|---|---|---|---|---|---|---|---|
| | Estimated A.M.&P.M.[b] | H&J[c] | Oppenlander[d] | Regression[e] | H&J[f] | Oppenlander[d] | Winfrey[g] |
| 5 | 31.08 | 23.40 | - | 5.06 | 4.80 | - | - |
| 10 | 15.54 | 11.70 | - | 4.86 | 3.69 | - | 5.19 |
| 15 | 10.36 | 7.80 | - | 4.54 | 3.10 | - | 4.53 |
| 20 | 7.77 | 5.85 | - | 4.10 | 2.78 | - | 4.22 |
| 25 | 6.22 | 4.68 | - | 3.58 | 2.57 | - | 4.04 |
| 30 | 5.18 | 3.90 | 4.92 | 3.14 | 2.41 | 3.80 | 3.96 |
| 35 | 4.44 | 3.34 | 4.22 | 2.68 | 2.36 | 3.87 | 3.95 |
| 40 | 3.89 | 2.93 | 3.69 | 2.30 | 2.32 | 4.04 | 4.00 |
| 45 | 3.45 | 2.60 | 3.28 | 2.04 | 2.57 | 4.16 | 4.08 |
| 50 | 3.11 | 2.34 | 2.95 | 1.92 | 2.82 | 4.41 | 4.20 |
| 55 | 2.83 | 2.13 | 2.69 | 1.92 | 3.08 | 4.79 | 4.36 |
| 60 | 2.59 | 1.95 | 2.46 | 2.18 | 3.35 | 5.28 | 4.58 |
| 65 | 2.39 | 1.80 | 2.27 | 2.68 | 3.65 | - | 4.87 |
| 70 | 2.22 | - | 2.11 | 3.52 | - | - | 5.25 |

[a]All costs and tolls are expressed in terms of cents per mile, unless otherwise specified; the results of the present analysis are labelled as "A.M." to indicate October 13 (morning), "P.M." to indicate October 13 (afternoon), and "Regression" to indicate results from the speed-fuel consumption regression; the Haikalis and Joseph results are indicated as "H&J".

[b]Assumes a value of time of $1.55/vehicle/hour, at 86¢/passenger/hour and an occupancy of 1.8 passengers/vehicle.

[c]Assumes a value of time of $1.17/vehicle/hour, at 75¢ passenger/hour and an occupancy of 1.56 passengers/vehicle.

[d]Reflects the cost of vehicles on four-lane rural highways during the daytime.

[e]Includes fuel cost at 32¢/gallon and assumes from AASHO (23), that fuel costs are 50 percent of total vehicle operating cost.

[f]Includes fuel (excluding fuel tax), oil, tire, and maintenance costs.

[g]Includes fuel, tire, oil, maintenance, and depreciation costs incurred at uniform rates of speed on roadway of zero grade.

Table 12a (Continued)

| System Speed (miles/ hour) | Time Cost Component | | | | Operating Cost Component | | | | Optimal Toll | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Estimated | | Johnson[h] | | Estimated | | Johnson[i] | | Estimated | | Johnson | | Walters[j] | Walters[k] |
| | A.M. | P.M. | | | A.M. | P.M. | | | A.M. | P.M. | | | | |
| 5 | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 10 | - | - | ∞ | ∞ | - | - | ∞ | ∞ | - | - | ∞ | ∞ | - | - |
| 15 | - | - | 3.80 | 13.46 | - | - | 0.79 | 2.80 | - | - | 4.59 | 16.25 | - | - |
| 20 | - | - | 1.11 | 4.01 | - | - | 0.23 | 0.83 | - | - | 1.34 | 4.84 | 94.5 | 62.0 |
| 25 | - | - | 0.09 | 2.14 | - | - | 0.12 | 0.45 | - | - | 0.10 | 2.58 | 28.3 | 22.8 |
| 30 | ∞ | ∞ | 0.00 | 1.25 | ∞ | - | 0.00 | 0.26 | ∞ | - | 0.00 | 1.51 | 14.4 | 12.3 |
| 35 | 388.54 | 50.88 | - | 0.72 | 27.29 | ∞ | 0.00 | 0.15 | 415.83 | ∞ | 0.00 | 0.88 | 9.0 | 7.8 |
| 40 | 21.76 | 17.81 | - | 0.00 | 5.30 | 11.33 | 0.00 | 0.00 | 27.06 | 29.14 | 0.00 | 0.00 | - | - |
| 45 | 6.94 | 7.88 | - | - | 1.75 | 2.95 | 0.00 | 0.00 | 8.69 | 10.83 | 0.00 | 0.00 | - | - |
| 50 | 2.77 | 3.33 | - | - | 0.26 | 0.43 | 0.00 | 0.00 | 3.03 | 3.75 | 0.00 | 0.00 | - | - |
| 55 | 1.14 | 1.66 | - | - | -0.42 | -0.72 | 0.00 | 0.00 | 0.72 | 0.93 | 0.00 | 0.00 | - | - |
| 60 | 0.42 | 0.79 | - | - | -0.56 | -1.17 | 0.00 | 0.00 | -0.14 | -0.37 | 0.00 | 0.00 | - | - |
| 65 | 0.08 | 0.32 | - | - | -0.21 | -1.07 | 0.00 | 0.00 | -0.14 | -0.74 | 0.00 | 0.00 | - | - |
| 70 | - | - | - | - | - | - | 0.00 | 0.00 | - | - | 0.00 | 0.00 | - | - |

[h] Assumes a value of time of \$0.90/vehicle/hour, at 50¢/passenger/hour and an occupancy of 1.8 passengers/vehicle.

[i] Based upon cost estimates of Haikalis and Joseph (47).

[j] Applies to the Lincoln Tunnel in New York.

[k] Applies to the Merritt Parkway in New York.

the fact that it is a decreasing function of speed. Not only does the customary approach implicitly interpret the critical speed and the optimum speed to be equal, but it is also restricted to the generation of a system of positive tolls.

As depicted in Figure 19, the operating cost component schedule estimated by the present approach is characterized by the following pattern: the component is negative for levels of flow less than optimum flow ($q_m'$), such that its absolute value is increasing with flow levels increasing and less than (what the present analysis refers to as) the "optimizing" flow denoted by $q_m^*$, and its absolute value is decreasing with flow levels increasing and greater than optimizing flow but less than optimal flow; and, the component is positive and increasing for levels of flow increasing and greater than optimal flow. That is, the operating cost component decreases as the level of system flow approaches optimizing flow from above or below.

The optimal toll is the sum of the time cost component and the operating cost component. While the traditional approach, exemplified by the Johnson and Walters schedules, relates to the negatively sloped portion of the fundamental diagram by discouraging levels of flow greater than the critical level, it refrains from any policy pronouncements concerning the positively sloped portion of the diagram,

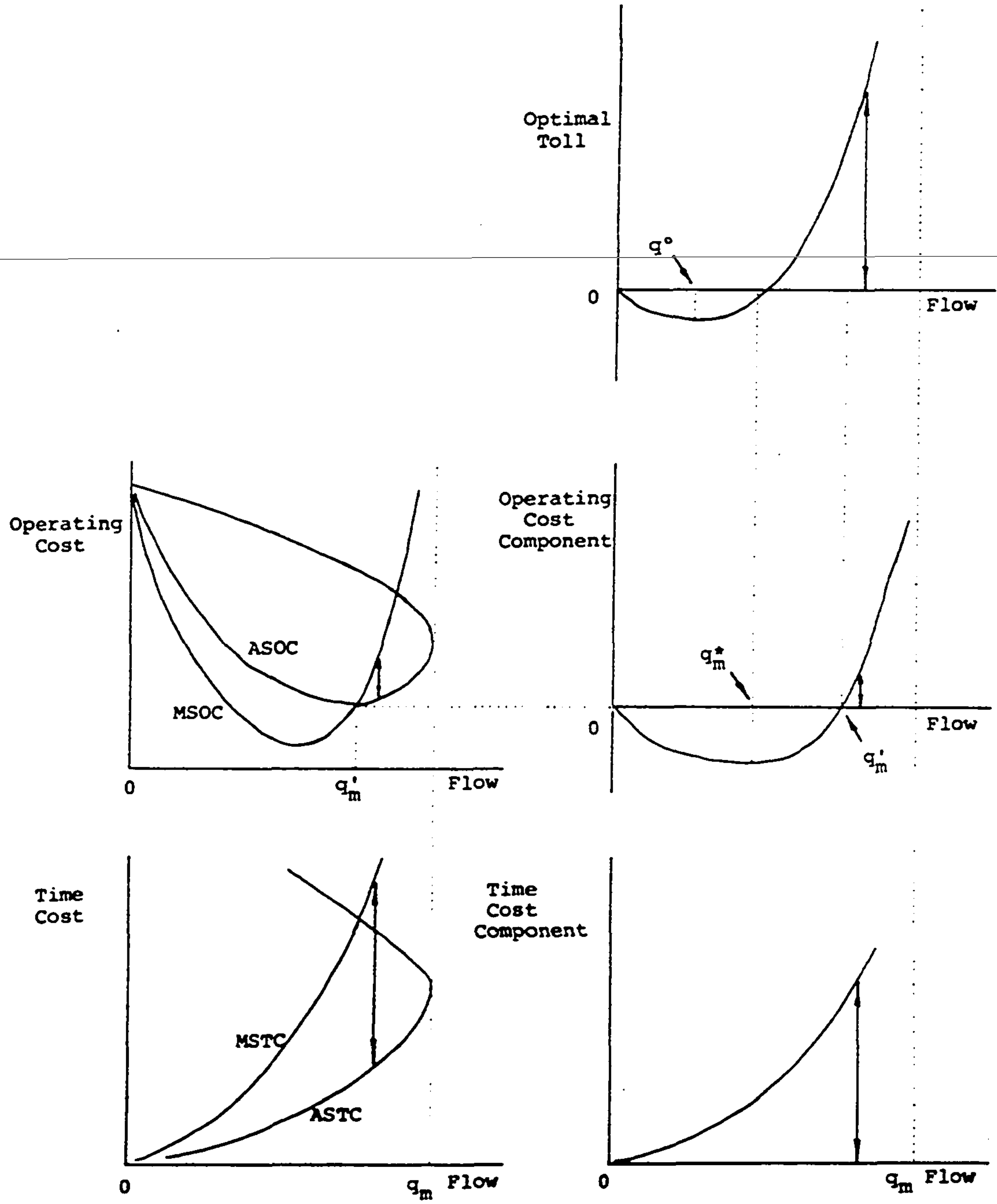


Figure 19. The basic model: aggregation of optimal toll components

consistent with relatively high rates of speed, vehicular operating cost, and accident risk: that is, it relates solely to the problem of assessing a "congestion" toll. The present approach, on the other hand, not only discourages levels of flow greater than optimizing flow -- by relying to an increasing extent upon the time cost component, relative to the operating cost component, as system flow increases -- but also discourages levels of flow less than optimizing flow: that is, it relates to the problem of assessing a "user" toll.

Three aspects of the optimal toll schedule estimated by the present analysis warrant consideration at this point. First, since the operating cost component is nonpositive for levels of flow less than or equal to optimum flow, there is the possibility that the optimal toll is nonpositive for some range of system speed. Referring to the optimal toll schedules of Table 12, it can be seen that the roadway geometrics, the vehicular traffic systems, and the monetary values assigned to the time cost and operating cost components are such as to generate nonpositive optimal tolls for a range of system speed during the morning and afternoon periods.

The second aspect concerns the relative magnitude of optimizing, optimum, and critical flow. Reference to the flow-operating cost functions portrayed in Figure 19 indicates that critical flow is the maximum level attained by the ASTC and ASOC functions. Optimum flow is defined by the intersection of MSOC with ASOC, and is consistent with an operating cost component of zero. Optimizing flow is the level of flow consistent with a minimum operating cost component, and is defined by equality of the slope of ASOC to the slope of MSOC.

A third aspect of the toll estimated by the present analysis relates to the fact that it increases as flow diverges from the level $q^{\circ}$, as seen in Figure 19. That is, it encourages the vehicular traffic system to operate at the level of flow at which the sum of the time cost and operating cost components of the optimal toll is minimized.

In the manner of Table 12, Figure 20 presents the estimated toll pattern as related to system speed. Optimum, critical, and optimizing speed are defined in a manner analogous to those levels of flow; and, $u^{\circ}$ is the level of system speed minimizing the sum of the time and operating cost components. By virtue of the fact that the relevant segment of the flow-time cost and the flow-operating cost functions ranges between zero flow (that is, speed equals $u_f$) and critical flow (that is, speed equals $u_m$), the toll schedules are not defined here for levels of system speed less than critical speed or greater than free speed.

Table 11 exhibited roadway toll schedules estimated under the assumption that the value of time for passenger cars is 2.59 cents per minute; these schedules were seen to be nonmonotonic with respect to system speed as well as negative for relatively high levels of system speed. Mohring (93, p. 2) comments regarding the use of 2.59 cents per minute as follows: "...it seems worthwhile to point out that a value of travel time is implicit in a driver's selection of a target speed."

Assuming that the target speed of a driver is the roadway free speed, and in deference to the fact that the traditionally estimated toll schedule monotonically approaches zero as system speed approaches free speed, the time cost components estimated by the present analysis are able to be adjusted to those shown in Table 12b by examining the absolute value of the ratio of the operating cost component to the time cost component at free speed, and multiplying the value of time by the amount of that absolute value. This adjustment indicates the value of time to be 9.64 and 11.34 cents per minute for the A.M. and P.M. time periods, respectively; Mohring estimates the value to be 12.30 cents per minute for target speed 60 miles per hour, and 113.03 cents per minute for target speed 70 miles per hour. The adjusted optimal toll schedule is, then, the aggregate of the adjusted time cost component and the operating cost component.

Two points related to the estimation procedure used in this study deserve consideration. First of all, the imposition of *a priori* restrictions upon the macroscopic (i.e., n=1)

Table 12b. Alternative toll schedules for 10-13-71

| System Speed (miles/ hour) | A.M. | | | | |
|---|---|---|---|---|---|
| | System Flow (vehicles/ hour) | Time Cost Component (cents/ mile) | Adjusted Time Cost Component (cents/ mile) | Operating Cost Component (cents/ mile) | Adjusted Optimal Toll (cents/ mile) |
| 65 | 195 | 0.076 | 0.281 | -0.210 | 0.070 |
| 60 | 660 | 0.418 | 1.556 | -0.556 | 1.001 |
| 55 | 1045 | 1.143 | 4.256 | -0.420 | 3.837 |
| 50 | 1350 | 2.768 | 10.307 | 0.260 | 10.568 |
| 45 | 1575 | 6.937 | 25.830 | 1.754 | 27.584 |
| 40 | 1720 | 21.764 | 81.042 | 5.298 | 86.339 |
| 35 | 1785 | 388.537 | 1446.788 | 27.288 | 1474.077 |
| 30 | 1770 | ∞ | ∞ | ∞ | ∞ |

| | | PM | | |
|---|---|---|---|---|
| System Flow (vehicles/ hour) | Time Cost Component (cents/ mile) | Adjusted Time Cost Component (cents/ mile) | Operating Cost Component (cents/ mile) | Adjusted Optimal Toll (cents/ mile) |
| 455 | 0.324 | 1.421 | -1.068 | 0.352 |
| 720 | 0.793 | 3.471 | -1.168 | 2.303 |
| 935 | 1.658 | 7.261 | -0.724 | 6.536 |
| 1100 | 3.326 | 14.566 | 0.426 | 14.991 |
| 1260 | 7.881 | 34.513 | 2.952 | 37.466 |
| 1320 | 17.813 | 78.010 | 11.328 | 89.338 |
| 1330 | 50.885 | 222.844 | ∞ | ∞ |
| 1290 | ∞ | ∞ | | |

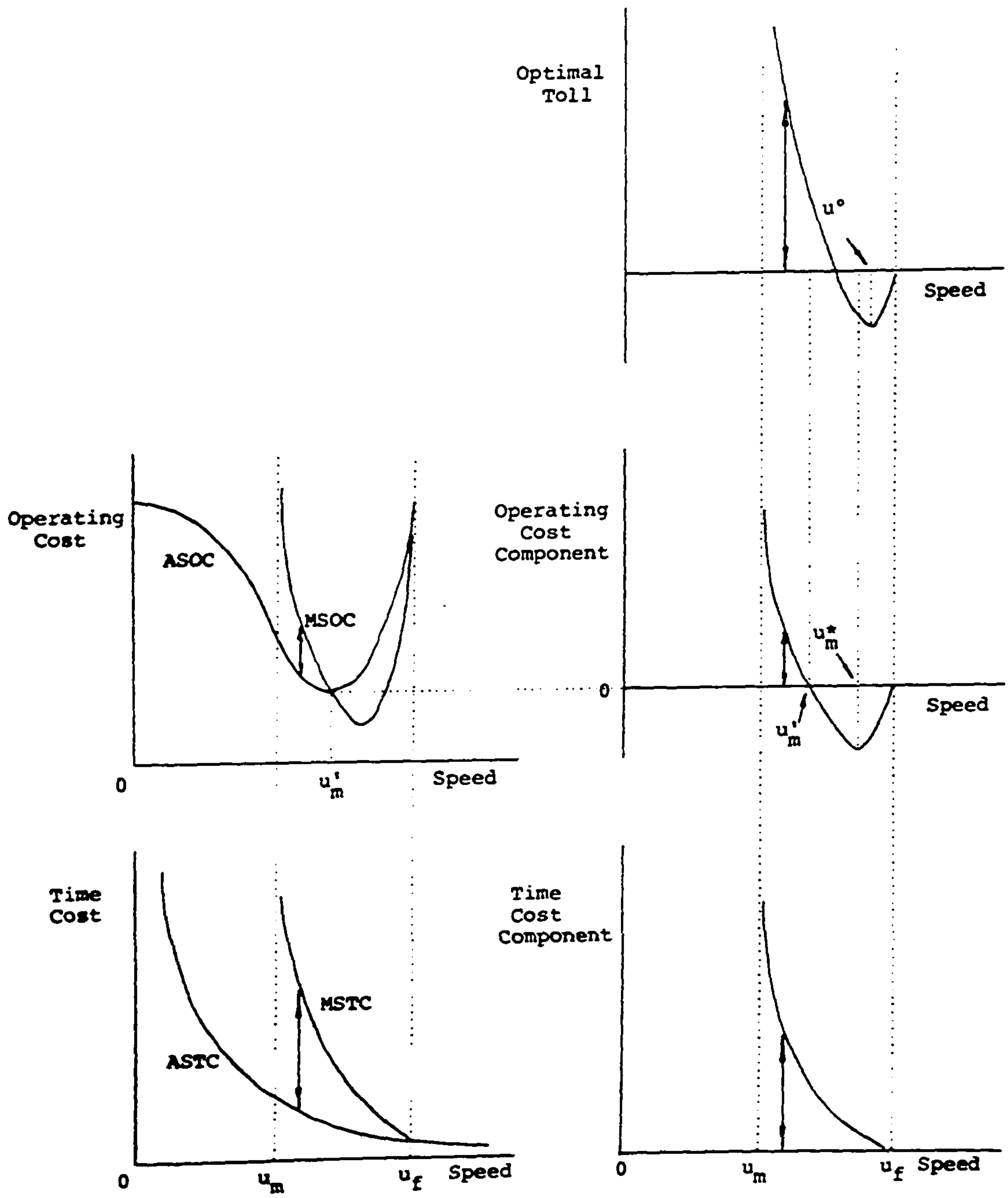


Figure 20. The basic model with respect to system speed

as well as microscopic (i.e., $b_2 = b_6 = 0$) regression analyses possibly precluded determination of the "best" functional relation among the variables. It was felt, though, that the satisfaction of these restrictions was more relevant to the analysis than were, for example, high multiple correlation coefficients.

Secondly, as a result of *a priori* theoretical considerations, the speed-acceleration noise equation was estimated by application of the general linear regression model with the restriction that the coefficient associated with the first-order term be equal to zero. The estimation produced $\sigma_{max}$, the intercept value, which was, in turn, able to be substituted into the coefficient associated with either the second- or third-order term in order to predict $u_f$. Analogous comments are applicable to the speed-fuel consumption equation. The argument might be levied, that these latter two equations should have been considered via a restricted estimation procedure. However, even though measurement error might have generated nonexact regression coefficients (i.e., $u_f$ predicted by $b_3$ being unequal to $u_f$ predicted by $b_4$), it was felt that application of the general linear regression model--rather than a restricted estimation model--would not do injustice to the estimation procedure, since the predicted roadway parameters $\sigma_{max}$, $F.C._{max}$, and $u_f$ reside outside the range of observable roadway data.

# CHAPTER VI: SUMMARY AND RECOMMENDATIONS

## Introduction

Given the premise that the force of self-rationing is inadequate to guarantee efficient use of urban roadway and that a pricing mechanism can be appropriate in immediate urban problems, a pricing model has been constructed and utilized to generate a user toll schedule. Based upon the fundamental diagram of road traffic and upon the recently introduced concept of acceleration noise, the model has the capacity to generate user toll schedules for sections of urban roadway, given a minimal amount of vehicular traffic system data.

## Consideration of the Data Analysis, and Extensions of the Model

An obvious criticism of the macroscopic and microscopic data employed by the analysis is that both types relate solely to periods of peak facility usage. Therefore, as is evident in the concentration-speed model (Figure 13), the speed-acceleration model (Figure 16), and the speed-fuel consumption model (Figure 17), there is a notable absence of observed traffic data for relatively high levels of system speed. Yet, in spite of this characteristic of the data, the theoretical link between the macroscopic and microscopic data (Figure 9) suggested by an *a priori* investigation of the analogy between vehicular traffic flow and fluid flow, was confirmed by the statistical analysis; a comparison (Table 6) of the levels of critical and optimum speed predicted by the two sets of data showed them to exhibit considerable consistency. The availability of data reflecting nonpeak as well as peak facility usage could be expected to have enhanced this consistency of predicted roadway parameters.

The basic model was specified in a form (that is, with n = 1 in Equations 17-22 and 31-34) consonant with the Greenshields assumption of a linear relationship between vehicular concentration and speed. An obvious extension of the present analysis is to investigate the predictive capability of the model specified in forms consistent with nonlinear concentration-speed relationships.

By virtue of the nontraditional nature of the operating cost component schedule estimated by the present analysis, the toll schedule was seen to be a nonmonotone function of traffic system flow; however, it was seen to monotonically increase as system flow increased from that level consistent with speed level $u^\circ$, and to monotonically decrease as system flow decreased from that level consistent with $u^\circ$ (Figure 19). The latter attribute results from the fact that the operating cost component is herein assumed to be a function solely of vehicular flow; whereas, without exception, other studies have assumed the operating cost component to be a function of time as well as vehicular flow in the sense that they are designed to capture the change in operating cost incurred by a change in travel time generated by a change in flow -- that is, to capture the incremental unit of time of operating cost incurred over a roadway section, resulting from a flow variation over that section.

The operating cost component estimated by the present analysis increases as system flow falls from the level consistent with optimizing speed, in order to account for the cost generated by relatively low levels of concentration: the allowance of vehicular operation at relatively high levels of speed, which are associated in a technological sense with a higher consumption rate of fuel, oil, etc. per unit distance. The obvious extension of the model to incorporate the direct and opportunity costs associated with the risk of vehicular accident supports this sort of operating cost schedule.

A review of the engineering literature related to vehicular traffic accident research revealed three general approaches to the topic: first, a preponderance of historical data relating vehicular accident rates to vehicular flow, concentration, and speed; second, an investigation by Treiterer (138) relating the physical laws of vehicular motion to the fundamental diagram of road traffic; and third, an investigation by Heimbach and Vick (52) relating vehicular accident potential to acceleration noise. While the last two independent approaches were found to be mathematically tractable and easily synthesized into the basic model (Figure 9), they were found to be presently lacking in empirical verification of the nature required for such a synthesis.

It is a sensible hypothesis that traffic accidents are, as vehicular operating costs, the result of erratic traffic system operation; such an assumption offers appeal to the Heimbach-Vick approach. Empirical confirmation of an accident-risk cost theory based upon the speed-acceleration noise model would lend support to the form of the operating cost component schedule estimated by the present analysis, for just as a user toll should account for the accident-risk cost associated with relatively high levels of speed and low levels of concentration, so should it account for analogous operating costs.

A second cost component not considered here, which future research might find to be easily accommodated by the speed-acceleration noise model, is the service cost of driver comfort and convenience (88, 99). Although such a cost is relatively intangible, "it can be assumed that this cost item increases at low speeds (driver impatience) and at high speeds (driver tension), whereas at intermediate speeds the service cost is minimized in the region of driver satisfaction" (99, p. 80).

An avenue of possible engineering inquiry relates to the use of level-of-service measures other than acceleration noise to capture the operating cost, accident-risk cost, and service cost components of the user toll (64, 154): prominent candidates include the Greenshields index (defined in Chapter IV, above) and mean velocity gradient (defined as the ratio of acceleration noise to mean velocity).

Finally, two general considerations of user tolls deserve comment. First of all, as with other studies of toll schedule estimation, one is in a position to merely surmise with respect to the reaction of vehicular traffic to the estimated toll schedule, in the sense of the roadway price achieving an equilibrium level.

And, secondly, while it is maintained that the present approach enhances the administrative feasibility of employing urban roadway pricing, the degree of political feasibility of vehicular traffic tolls remains low in comparison to that of such schemes as increased parking rates, improved public transit service, staggered work hours, and the like. One feasible way of making such a toll more politically palatable is to impose it in the form of a block schedule. Whereas such a pricing mechanism is subject to error in the sense of over- or underestimating the optimal schedule within a block, it is able to be levied in a manner tempered by political and institutional authority in the light of non-economic considerations as are, analogously, the rate schedules of the electric and gas industries.

BIBLIOGRAPHY

1. Baumol, William J. Economic theory and operations analysis. Englewood Cliffs, New Jersey, Prentice-Hall, Inc. 1965.

2. Becker, Gary S. A theory of the allocation of time. Economic Journal 75: 493-517. 1965.

3. Bèckmann, Martin, McGuire, C. B. and Winsten, Christopher B. Studies in the economics of transportation. New Haven, Yale University Press. 1956.

4. Beesley, M. E. 'Motorways in London' and transport planning. Environment and Planning 2: 137-151. 1970.

5. Beesley, M. E. The value of time spent in travelling: Some new evidence. Economica 32: 174-185. 1965.

6. Beesley, M. E. and Kain, J. F. Urban form, car ownership and public policy: An appraisal of Traffic in Towns. Urban Studies 1: 174-203. 1964.

7. Beesley, M. E. and Roth, G. J. Restraint of traffic in congested areas. The Town Planning Review 33: 184-196. 1962.

8. Bellomo, Salvatore J. and Provost, Steven C. Two procedures to improve the economic evaluation of alternative highway systems. Highway Research Record 224: 44-53. 1968.

9. Bergendahl, Göran. A combined linear and dynamic programming model for interdependent road investment planning. Transportation Research 3: 211-228. 1969.

10. Bone, A. J. Travel-time and gasoline-consumption studies in Boston. Highway Research Board Proceedings 31: 440-456. 1952.

11. Bostick, T. A. and Todd, T. R. Travel characteristics of persons living in larger cities. Highway Research Record 106: 52-57. 1966.

12. Botzow, Hermann. An empirical method for estimating auto commuting costs. Highway Research Record 197: 56-69. 1967.

13. Buchanan, James M. The pricing of highway services. National Tax Journal 5: 97-106. 1952.

14. Capelle, Donald G. An investigation of acceleration noise as a measure of freeway level of service. Unpublished Ph.D. dissertation. College Station, Texas, Library, Texas A&M University. 1966.

15. Carlin, Alan and Park, R. E. Marginal cost pricing of airport runway capacity. The American Economic Review 60: 310-319. 1970.

16. Carmichael, Thomas J. and Haley, Charles E. A study of vehicle, roadway, and traffic relationships by means of statistical instruments. Highway Research Board Proceedings 30: 282-296. 1950.

17. Carroll, J. Douglas. Towards a national policy for urban transportation. In United Transportation Union, 1970 conference on mass transportation, University of Chicago. Vol. 4. Chicago, Illinois, United Transportation Union Mass Transportation Series. 1970.

18. Carstens, R. L. A model for estimating transit usage in cities in Iowa. Highway Research Record 213: 42-49. 1968.

19. Chandler, R. E., Herman, R. and Montroll, E. W. Traffic dynamics: Studies in car following. Operations Research 6: 165-184. 1958.

20. Claffey, Paul J. Characteristics of passenger-car travel on toll roads and comparable free roads for highway user benefit studies. Public Roads 31: 167-176. 1961.

21. Coase, R. H. Price and output policy of state enterprise: A comment. Economic Journal 55: 112-113. 1945.

22. Coase, R. H. The theory of public utility pricing and its application. The Bell Journal of Economics and Management Science 1: 113-128. 1970.

23. Committee on Planning and Design Policies, Road User Benefit Analysis for Highway Improvements. Washington, D.C., American Association of State Highway Officials. 1960.

24. Creighton, Roger L., Gooding, David I., Hemmens, George C. and Fidler, Jere E. Optimum investment in two-mode transportation systems. Highway Research Record 47: 23-45. 1964.

25. Curtin, John F. Effect of fares on transit riding. Highway Research Record 213: 8-18. 1968.

26. Dafermos, Stella and Sparrow, F. T. Optimal resource allocation and toll patterns in user-optimized transport networks. Journal of Transport Economics and Policy 5: 184-200. 1971.

27. DeWeille, Jan. Quantification of road user savings. Washington, D.C., International Bank for Reconstruction and Development. 1967.

28. Drake, J., Schofer, J. and May, A. A statistical analysis of speed-density hypotheses. In Edie, Leslie C., Herman, Robert and Rothery, Richard, ed. Vehicular traffic science. New York, New York, American Elsevier Publishing Company. 1967.

29. Drew, Donald R. Traffic flow theory and control. New York, New York, McGraw-Hill Book Company, Inc. 1968.

30. Drew, Donald R. and Keese, Charles J. Freeway level of service as influenced by volume and capacity characteristics. Highway Research Record 99: 1-39. 1965.

31. Drew, Donald R., Dudek, Conrad L. and Keese, Charles J. Freeway level of service as described by an energy-acceleration noise model. Highway Research Record 162: 30-85. 1967.

32. Dréze, Jacques H. Some postwar contributions of French economists to theory and public policy, with special emphasis on problems of resource allocation. The American Economic Review Supplement 54: 4-64. 1964.

33. Duncan, W. J., Thom, A. S. and Young, A. D. Mechanics of fluids. Second edition. New York, New York, American Elsevier Publishing Company, Inc. 1970.

34. Eckstein, Otto. Public finance. Englewood Cliffs, New Jersey, Prentice-Hall, Inc. 1967.

35. Edie, Leslie C. and Foote, Robert S. Traffic flow in tunnels. Highway Research Board Proceedings 37: 334-344. 1958.

36. Fitch, Lyle C. and Associates. Urban transportation and public policy. San Francisco, California, Chandler Publishing Company. 1964.

37. Ganz, Alexander. Emerging patterns of urban growth and travel. Highway Research Record 229: 21-37. 1968.

38. Garfield, Paul J. and Lovejoy, Wallace F. Public utility economics. Englewood Cliffs, New Jersey, Prentice-Hall, Inc. 1964.

39. Gibbins, John W. and Proctor, Albert. Economic costs of traffic congestion. Highway Research Bulletin 86: 1-25. 1954.

40. Goldin, Kenneth D. Three aspects of highway efficiency: Amount, quality and price. Journal of Transport Economics and Policy 2: 349-366. 1968.

41. Greenberg, Harold. An analysis of traffic flow. Operations Research 7: 79-85. 1959.

42. Greenberg, Harold and Daou, Arthur. The control of traffic flow to increase the flow. Operations Research 8: 524-532. 1960.

43. Greenshields, Bruce D. The quality of traffic flow. In Greenshields, Bruce D., George, Henry P., Guerin, Neil S., Palmer, M. Ross and Underwood, Robin T. The quality and theory of traffic flow. New Haven, Connecticut, Bureau of Highway Traffic, Yale University. 1961.

44. Greenshields, Bruce D. A study of traffic capacity. Highway Research Board Proceedings 14: 448-477. 1934.

45. Gwilliam, K. M. Transport and public policy. London, George Allen and Unwin Ltd. 1964.

46. Haight, Frank A. Mathematical theories of traffic flow. New York, New York, Academic Press. 1963.

47. Haikalis, George and Joseph, Hyman. Economic evaluation of traffic networks. Highway Research Board Bulletin 306: 39-63. 1961.

48. Hall, Edward M. and George, Stephen, Jr. Travel time -- an effective measure of congestion and level of service. Highway Research Board Porceedings 38: 511-529. 1959.

49. Haney, Dan G. Use of two concepts of the value of time. Highway Research Record 12: 1-18. 1963.

50. Hayes, John J. Some considerations of vehicular density on urban freeways. Highway Research Record 99: 59-80. 1965.

51. Hedges, Charles A. An evaluation of commuter transportation alternatives. Highway Research Record 296: 12-46. 1969.

52. Heimbach, Clinton L. and Vick, Harold D. Relating change of highway speed per unit of time to motor vehicle accident rates. Highway Research Record 225: 44-65. 1968.

53. Henderson, James M. and Quandt, Richard E. Microeconomic theory: A mathematical approach. Second edition. New York, New York, McGraw-Hill Book Company. 1971.

54. Herman, Robert and Potts, R. B. Single-lane traffic theory and experiment. In Herman, Robert, ed. Theory of traffic flow. New York, New York, Elsevier Publishing Company. 1961.

55. Herman, Robert, Montroll, E. W., Potts, R. B. and Rothery, R. W. Traffic dynamics: Analysis of stability in car following. Operations Research 7: 86-106. 1959.

56. Hewitt, J. The calculation of congestion taxes on roads. Economica 31: 72-81. 1964.

57. Highway Research Board. Highway capacity manual. Highway Research Board Special Report 87. 1965.

58. Highway Research Board. Traffic surveillance and means of communicating with drivers: Interim report. Highway Research Board National Cooperative Highway Research Program Report 9. 1964.

59. Highway Research Board. Running costs of motor vehicles as affected by highway design: Interim report. Highway Research Board National Cooperative Highway Research Program Report 13. 1965.

60. Highway Research Board. Surveillance methods and ways and means of communicating with drivers. Highway Research Board National Cooperative Highway Research Program Report 28. 1966.

61. Highway Research Board. National survey of transportation attitudes and behavior: Phase I, summary report. Highway Research Board National Cooperative Highway Research Program Report 49. 1968.

62. Highway Research Board. Research needs in highway transportation. Highway Research Board National Cooperative Highway Research Program Report 55. 1968.

63. Highway Research Board. Running costs of motor vehicles as affected by road design and traffic. Highway Research Board National Cooperative Highway Research Program Report 111. 1971.

64. Highway Research Board. Optimizing flow in existing street networks. Highway Research Board National Cooperative Highway Research Program Report 113. 1971.

65. Hirshleifer, Jack. Peak loads and efficient pricing: Comment. Quarterly Journal of Economics 72: 451-462. 1958.

66. Huber, Matthew J. Effect of temporary bridge on parkway performance. Highway Research Board Bulletin 167: 63-74. 1957.

67. Humphreys, Jack B. Effect of trucks on the urban freeway. Highway Research Record 308: 62-75. 1970.

68. Johnson, M. Bruce. On the economics of road congestion. Econometrica 32: 137-150. 1964.

69. Johnson, M. Bruce. Travel time and the price of leisure. The Western Economic Journal 4: 135-145. 1966.

70. Johnston, J. Econometric methods. New York, New York, McGraw-Hill Book Company. 1963.

71. Johnston, J. Statistical cost analysis. New York, New York, McGraw-Hill Book Company. 1960.

72. Jones, Trevor R. and Potts, Renfrey B. The measurement of acceleration noise -- A traffic parameter. Operations Research 10: 745-763. 1962.

73. Kafoglis, Milton Z. Highway policy and external economies. National Tax Journal 16: 68-80. 1963.

74. Kain, John F. A contribution to the urban transportation debate: An econometric model of urban residential and travel behavior. The Review of Economics and Statistics 46: 55-64. 1964.

75. Kasoff, Mark J. The quality of service and transit use. Traffic Quarterly 24: 107-119. 1970.

76. Kuhn, Tillo E. Public enterprise economics and transport problems. Berkeley and Los Angeles, California, University of California Press. 1962.

77. Lansing, John B. How people perceive the cost of the journey to work. Highway Research Record 197: 44-55. 1967.

78. Lassow, William. Effect of the fare increase of July 1966 on the number of passengers carried on the New York City Transit System. Highway Research Record 213: 1-7. 1968.

79. Lerner, Abba P. Everybody's business. East Lansing, Michigan, Michigan State University Press. 1961.

80. Lichfield, Nathaniel and Chapman, Honor. The urban transport problem and modal choice. Journal of Transport Economics and Policy 5: 247-266. 1971.

81. Locklin, D. Philip. Economics of transportation. Homewood, Illinois, Richard D. Irwin, Inc. 1966.

82. Luna, Charles. The UTU handbook of transportation in America. New York, New York, Popular Library. 1971.

83. May, Adolf D. and Athol, Patrick. Development and evaluation of Congress Street Expressway pilot detection system. Highway Research Record 21: 48-68. 1963.

84. May, Adolf D. and Wagner, Frederick A. Headway characteristics and interrelationships of fundamental characteristics of traffic flow. Highway Research Board Proceedings 39: 524-547. 1960.

85. McKie, James W. Regulation and the free market: The problem of boundaries. The Bell Journal of Economics and Management Science 1: 6-26. 1970.

86. Meade, J. E. and Fleming, J. M. Price and output policy of state enterprise: A symposium. Economic Journal 54: 321-339. 1944.

87. Meiburg, Charles O. An economic analysis of highway services. Quarterly Journal of Economics 77: 648-656. 1963.

88. Mercellis, Jack C. An economic evaluation of traffic movement at various speeds. Highway Research Record 35: 18-40. 1963.

89. Meyer, John R. Statistical cost functions: Some methodological aspects of statistical costing as illustrated by the determination of rail passenger costs. The American Economic Review Papers and Proceedings 48: 209-222. 1958.

90. Meyer, J. R., Kain, J. F. and Wohl, M. The urban transportation problem. Cambridge, Massachusetts, Harvard University Press. 1965.

91. Meyer, John R., Peck, Merton J., Stenason, John and Zwick, Charles. The economics of competition in the transportation industries. Cambridge, Massachusetts, Harvard University Press. 1959.

92. Mishan, E. J. Interpretation of the benefits of private transport. Journal of Transport Economics and Policy 1: 184-189. 1967.

93. Mohring, Herbert. Relation between optimum congestion tolls and present highway user charges. Highway Research Record 47: 1-14.

94. Mohring, Herbert. Urban highway investments. In Dorfman, Robert, ed. Measuring benefits of government investments. Pp. 231-291. Washington, D.C., The Brookings Institution. 1965.

95. Moses, Leon N. and Williamson, Harold F., Jr. Value of time, choice of mode, and the subsidy issue in urban transportation. The Journal of Political Economy 71: 247-264.

96. Nelson, James C. The pricings of highway, waterway, and airway facilities. The American Economic Review Papers and Proceedings 52: 426-435. 1962.

97. Olcott, Edward S. The influence of vehicular speed and spacing on tunnel capacity. Operations Research 3: 147-167. 1955.

98. Oort, C. J. The evaluation of travelling time. Journal of Transport Economics and Policy 3: 279-286. 1969.

99. Oppenlander, J. C. A theory on vehicular speed regulation. Highway Research Board Bulletin 341: 77-91. 1962.

100. Owen, John D. The value of commuter speed. Western Economic Journal 7: 164-172. 1969.

101. Palmer, M. Ross. The development of traffic congestion. In Greenshields, Bruce C., George, Henry P., Guerin, Neil S., Palmer, M. Ross and Underwood, Robin T. The quality and theory of traffic flow. New Haven, Connecticut, Bureau of Highway Traffic, Yale University. 1961.

102. Paul, M. E. Covering costs by receipts. Oxford University Institute of Statistics Bulletin 22: 299-311. 1960.

103. Phillips, Charles F. Jr. The economics of regulation theory and practice in the transportation and public utility industries. Homewood, Illinois, Richard D. Irwin, Inc. 1965.

104. Platt, Fletcher N. A proposed index for the level of traffic service. Traffic Engineering 34: 21-26. 1963.

105. Ponsonby, G. J. The problem of the peak, with special reference to road passenger transport. Economic Journal 68: 74-88. 1958.

106. Pressman, Israel. A mathematical formulation of the peak-load pricing problem. The Bell Journal of Economics and Management Science 1: 304-326. 1970.

107. Quarmby, D. A. Choice of travel mode for the journey to work. Journal of Transport Economics and Policy 1: 273-314. 1967.

108. Reuss, Henry S. The federal government and coordinated transportation. Highway Research Record 183: 1-6. 1967.

109. Roth, G. J. An economic approach to traffic congestion. In Almond, Joyce, ed. Proceedings of the second international symposium on the theory of road traffic flow, London, 1963. Pp. 304-316. Paris, The Organisation for Economic Co-operation and Development. 1965.

110. Roth, Gabriel J. Traffic congestion as a source of revenue. Traffic Quarterly 24: 175-190. 1970.

111. Ruggles, Nancy. Recent developments in the theory of marginal cost pricing. Review of Economic Studies 17: 107-126. 1949-50.

112. Ruggles, Nancy. The welfare basis of the marginal cost pricing principle. Review of Economic Studies 17: 29-46. 1949-50.

113. Sabersky, Rolf H., Acosta, Allan J. and Hauptmann, Edward G. Fluid flow. Second edition. New York, New York, The Macmillan Company. 1971.

114. Samuelson, Paul A. Aspects of public expenditure theories. Review of Economics and Statistics 40: 332-338. 1958.

115. Sawhill, Roy B. and Crandall, Keith C. Some measurable qualities of traffic service influenced by freeways. Highway Research Record 49: 3-63. 1964.

116. Sawhill, Roy B., Matteson, James H. and Hall, Jerome W. Vehicle characteristics of fuel and travel time on urban arterials and freeways. Highway Research Record 314: 41-59. 1970.

117. Seely, Fred B., Ensign, Newton E. and Jones, Paul G. Analytical mechanics for engineers. Fifth edition. New York, New York, John Wiley and Sons, Inc. 1958.

118. Segelhorst, Elbert W. Transit validation for city centers. Journal of Transport Economics and Policy 5: 28-39. 1971.

119. Sharp, C. Congestion and welfare -- An examination of the case for a congestion tax. Economic Journal 76: 806-817. 1966.

120. Sharp, C. H. The choice between cars and buses on urban roads. Journal of Transport Economics and Policy 1: 104-111. 1967.

121. Sharp, Clifford. Problems of urban passenger transport with special reference to Leicester. Leicester, England, Leicester University Press. 1967.

122. Sherman, Roger. A private bias in transit choice. The American Economic Review 57: 1211-1217. 1967.

123. Smeed, R. J. Road pricing: The economic and technical possibilities. London, Her Majesty's Stationery Office. 1964.

124. Smeed, R. J. Traffic studies and urban congestion. Journal of Tranport Economics and Policy 2: 33-70. 1968.

125. Smerk, George M. Urban transportation: The federal role. Bloomington, Indiana, Indiana University. 1965.

126. Soberman, Richard M. and Clark, George A. A general-purpose model for motor vehicle operating costs. Highway Research Record 314: 60-71. 1970.

127. St. Clair, G. P. Congestion tolls -- An Engineer's viewpoint. Highway Research Record 47: 66-112. 1964.

128. Starkie, D. N. M. Modal split and the value of time: A note on idle-time. Journal of Transport Economics and Policy 5: 216-220. 1971.

129. Steiner, Peter O. Peak loads and efficient pricing. Quarterly Journal of Economics 71: 585-610. 1957.

130. Stollsteimer, J. F., Bressler, R. G. and Boles, J. N. Cost functions from cross-section data -- Fact or fantasy? Agricultural Economics Research 13: 79-88. 1961.

131. Streeter, Victor L. Fluid mechanics. New York, New York, McGraw-Hill Book Company, Inc. 1958.

132. Strotz, Robert H. Principles of urban transportation pricing. Highway Research Record 47: 113-121. 1964.

133. Thomas, Thomas C. and Thompson, Gordon I. The value of time for commuting motorists as a function of their income level and amount of time saved. Highway Research Record 314: 1-19. 1970.

134. Thomson, J. M. An evaluation of two proposals for traffic restraint in central London. Journal of the Royal Statistical Society, Series A, 130: 327-367. 1967.

135. Tipping, D. G. Time savings in transport studies. Economic Journal 78: 843-854. 1968.

136. Torres, J. F. Acceleration noise, power spectra, and other statistics derived from instrumented vehicle measurements under freeday driving conditions. Highway Research Record 308: 13-33. 1970.

137. Traffic: Time for some speed. The Economist 217: 1182. 1965.

138. Treiterer, Joseph. Improvement of traffic flow and safety by longitudinal control. Transportation Research 1: 231-251. 1967.

139. Turvey, Ralph. On divergences between social cost and private cost. Economica 30: 309-313. 1963.

140. Tyndall, David G. The relative merits of average cost pricing, marginal cost pricing, and price discrimination. Quarterly Journal of Economics 65: 342-372. 1951.

141. Vaswani, Ram. Value of automobile transit time in highway planning. Highway Research Board Proceedings 37: 58-71. 1958.

142. Vickrey, William. Congestion charges and welfare. Journal of Transport Economics and Policy 2: 107-118. 1968.

143. Vickrey, William. Notes on urban transportation issues. Unpublished mimeographed paper. New York, New York, Department of Economics, Columbia University. 1971.

144. Vickrey, William. Optimization of traffic and facilities. Journal of Transport Economics and Policy 1: 123-136. 1967.

145. Vickrey, William. Pricing as a tool in coordination of local transportation. In Transportation economics. Pp. 275-296. New York, New York, National Bureau of Economic Research. 1965.

146. Vickrey, William. Pricing in urban and suburban transport. The American Economic Review Papers and Proceedings 53: 452-465. 1963.

147. Vickrey, William. Some implications of marginal cost pricing for public utilities. The American Economic Review Papers and Proceedings 45: 605-620. 1955.

148. Vickrey, William. Some objections to marginal-cost pricing. Journal of Political Economy 56: 218-238. 1948.

149. Walters, A. A. The economics of road user charges. Baltimore, Maryland, The Johns Hopkins Press. 1968.

150. Walters, A. A. Empirical evidence on optimal motor taxes for the United Kingdom. Applied Statistics 10: 157-169. 1961.

151. Walters, A. A. Production and cost functions: An econometric survey. Econometrica 31: 1-66. 1963.

152. Walters, A. A. The theory and measurement of private and social cost of highway congestion. Econometrica 29: 676-699. 1961.

153. Whitaker, Stephen. Introduction to fluid mechanics. Englewood Cliffs, New Jersey, Prentice-Hall, Inc. 1968.

154. Whitson, Robert H., Swift, Gilbert and Milberger, Lionel J. Use of instrumented vehicle in evaluating arterial street operations. An unpublished draft prepared for presentation at the 50th annual meeting of the Highway Research Board. College Station, Texas, Texas Transportation Institute, Texas A&M University. January, 1971.

155. Williams, Penelope M. Low fares and the urban transport problem. Urban Studies 6: 83-92. 1969.

156. Winch, David M. The economics of highway planning. Toronto, University of Toronto Press. 1963.

157. Winfrey, Robley. Economic analysis for highways. Scranton, Pennsylvania, International Textbook Company. 1969.

158. Wingo, Lowdon, Jr. Transportation and urban land. Washington, D.C., Resources for the future. 1964.

159. Wiseman, J. The theory of public utility price -- An empty box. Oxford Economic Papers 9: 56-74. 1957.

160. Wohl, Martin. Congestion toll pricing for public transport facilities. Highway Research Record 314: 20-31. 1970.

161. Wohl, Martin and Martin, Brian V. Traffic system analysis for engineers and planners. New York, New York, McGraw-Hill Book Company. 1967.

162. Zettel, Richard M. and Carll, Richard R. The basic theory of efficiency tolls. Highway Research Record 47: 46-65. 1964.

163. Ziegler, Charles M. Our highway needs and how they can be financed. National Tax Association Proceedings 46: 80-88. 1953.

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# APPENDIX A

## Frequently-used Notation

$\alpha$ = kinetic-energy correction factor

$a, a_1, \ldots, a_7$ = coefficients

$a_{ave}$ = average acceleration of a vehicle unit during a trip

$a(t)$ = acceleration of a vehicle unit at time t

ASOC = average social operating cost

ASTC = average social time cost

$b, b_1, \ldots, b_8$ = coefficients

c = coefficient

e = internal energy

E = kinetic energy

$\varepsilon_{ASOC}$ = elasticity of average social operating cost

$\varepsilon_{ASTC}$ = elasticity of average social time cost

F.C. = total fuel consumption

$F.C._{max}$ = maximum fuel consumption

k = concentration

$k_j$ = jam concentration

$k_m$ = critical concentration

$k'_m$ = optimum concentration

MSOC = marginal social operating cost

MSTC = marginal social time cost

$\frac{mv^2}{2}$ = kinetic energy

| | |
|---|---|
| $n$ | = constant of proportionality |
| $\phi$ | = potential energy |
| $\Omega$ | = total energy |
| O.C. | = total operating cost |
| $O.C._{max}$ | = maximum operating cost |
| $q$ | = flow |
| $q_m$ | = critical flow |
| $q_m'$ | = optimum flow |
| $q_m^*$ | = optimizing flow |
| $\sigma$ | = total measured acceleration noise |
| $\sigma_I$ | = interaction acceleration noise |
| $\sigma_{max}$ | = maximum measured acceleration noise |
| $\sigma_{min}$ | = minimum measured acceleration noise |
| $\sigma_N$ | = natural acceleration noise |
| $t$ | = time |
| $t^{-1}$ | = speed |
| $u$ | = speed |
| $u_f$ | = free speed |
| $u_m$ | = critical speed |
| $u_m'$ | = optimum speed |
| $u_m^*$ | = optimizing speed |
| $u(t)$ | = speed of a vehicle unit at time t |

# APPENDIX B

## Macroscopic Data

| 10-13-71 A.M. | | 10-13-71 P.M. | | 10-19-71 A.M. | |
|---|---|---|---|---|---|
| Speed (miles/ hour) | Concentration (vehicles/ mile) | Speed (miles/ hour) | Concentration (vehicles/ mile) | Speed (miles/ hour) | Concentration (vehicles/ mile) |
| 50.00000 | 21.00000 | 81.00000 | 5.00000 | 27.00000 | 38.00000 |
| 47.00000 | 26.00000 | 79.00000 | 17.00000 | 35.00000 | 24.00000 |
| 44.00000 | 38.00000 | 58.00000 | 20.00000 | 43.00000 | 30.00000 |
| 41.00000 | 38.00000 | 49.00000 | 24.00000 | 37.00000 | 44.00000 |
| 41.00000 | 42.00000 | 47.00000 | 23.00000 | 36.00000 | 33.00000 |
| 37.00000 | 46.00000 | 41.00000 | 29.00000 | 39.00000 | 40.00000 |
| 39.00000 | 41.00000 | 49.00000 | 23.00000 | 34.00000 | 41.00000 |
| 47.00000 | 30.00000 | 51.00000 | 20.00000 | 32.00000 | 46.00000 |
| 59.00000 | 24.00000 | 45.00000 | 23.00000 | 26.00000 | 60.00000 |
| 58.00000 | 34.00000 | 50.00000 | 21.00000 | 22.00000 | 62.00000 |
| 39.00000 | 48.00000 | 61.00000 | 23.00000 | 20.00000 | 65.00000 |
| 34.00000 | 51.00000 | 56.00000 | 24.00000 | 20.00000 | 66.00000 |
| 32.00000 | 50.00000 | 49.00000 | 31.00000 | 20.00000 | 62.00000 |
| 31.00000 | 57.00000 | 44.00000 | 28.00000 | 15.00000 | 100.00000 |
| 41.00000 | 18.00000 | 54.00000 | 23.00000 | 10.00000 | 139.00000 |
| 53.00000 | 23.00000 | 48.00000 | 28.00000 | 7.00000 | 159.00000 |
| 44.00000 | 33.00000 | 41.00000 | 34.00000 | 8.00000 | 128.00000 |
| 45.00000 | 33.00000 | 41.00000 | 26.00000 | 11.00000 | 104.00000 |
| 45.00000 | 35.00000 | 42.00000 | 25.00000 | 14.00000 | 73.00000 |
| 45.00000 | 33.00000 | 46.00000 | 27.00000 | 21.00000 | 44.00000 |
| 45.00000 | 35.00000 | 45.00000 | 19.00000 | 31.00000 | 34.00000 |
| 47.00000 | 34.00000 | 50.00000 | 21.00000 | 34.00000 | 35.00000 |
| 55.00000 | 24.00000 | 47.00000 | 27.00000 | 37.00000 | 36.00000 |
| 57.00000 | 30.00000 | 48.00000 | 20.00000 | 36.00000 | 31.00000 |
| 56.00000 | 25.00000 | 54.00000 | 15.00000 | 39.00000 | 32.00000 |
| 62.00000 | 20.00000 | 57.00000 | 21.00000 | 38.00000 | 31.00000 |
| 68.00000 | 20.00000 | 48.00000 | 15.00000 | 41.00000 | 25.00000 |
| 51.00000 | 24.00000 | 54.00000 | 17.00000 | 40.00000 | 25.00000 |
| 42.00000 | 23.00000 | 52.00000 | 18.00000 | 41.00000 | 22.00000 |
| 41.00000 | 31.00000 | 49.00000 | 22.00000 | 43.00000 | 24.00000 |
| 40.00000 | 24.00000 | 55.00000 | 12.00000 | 39.00000 | 24.00000 |
| | | 68.00000 | 13.00000 | | |
| | | 57.00000 | 15.00000 | | |
| | | 46.00000 | 20.00000 | | |
| | | 47.00000 | 18.00000 | | |
| | | 56.00000 | 16.00000 | | |
| | | 46.00000 | 21.00000 | | |
| | | 48.00000 | 15.00000 | | |
| | | 55.00000 | 14.00000 | | |

# APPENDIX C

## Microscopic Data

Table 13. Microscopic data from moving vehicle studies (driver Welch)

| Moving Vehicle Study Number | Microscopic Subsystem Number | Mean Velocity (ft/sec) | Acceleration Noise (ft/sec/sec) | Fuel Consumption (miles/gal) |
|---|---|---|---|---|
| 526 | 2 | 40.4 | 1.84 | 12.61 |
| | 3 | 55.0 | 1.06 | 22.02 |
| 528 | 2 | 36.9 | 1.58 | 17.62 |
| | 3 | 47.9 | 1.20 | 19.50 |
| 530 | 2 | 33.5 | 1.96 | 11.60 |
| | 3 | 50.8 | 1.45 | 19.66 |
| 556 | 2 | 33.3 | 1.51 | 14.99 |
| | 3 | 54.1 | 1.12 | 20.38 |
| 558 | 2 | 27.6 | 1.99 | 11.30 |
| | 3 | 54.4 | 1.18 | 21.78 |
| 581 | 2 | 25.4 | 2.87 | 11.03 |
| | 3 | 29.7 | 1.97 | 11.08 |
| 583 | 2 | 24.0 | 2.13 | 11.27 |
| | 3 | 31.6 | 1.45 | 11.54 |
| 607 | 2 | 36.7 | 1.09 | 16.63 |
| | 3 | 40.0 | 1.75 | 18.89 |
| 609 | 2 | 65.1 | 1.55 | 24.57 |
| | 3 | 56.8 | 1.62 | 26.50 |
| 611 | 2 | 63.2 | 1.41 | 25.18 |
| | 3 | 69.0 | 1.44 | 19.49 |
| 1279 | 2 | 76.3 | 1.26 | 38.51 |
| | 3 | 69.5 | 1.43 | 34.12 |
| 1281 | 2 | 57.3 | 1.02 | 32.46 |
| | 3 | 57.2 | 1.31 | 28.81 |
| 1283 | 2 | 30.4 | 1.26 | 20.76 |
| | 3 | 41.3 | 1.71 | 23.27 |
| 1285 | 2 | 38.0 | 2.24 | 22.95 |
| | 3 | 46.0 | 1.68 | 21.59 |
| 1287 | 2 | 59.6 | 2.32 | 22.46 |
| | 3 | 60.7 | 1.58 | 25.31 |
| 1392 | 2 | 22.0 | 1.91 | 14.14 |
| | 3 | 46.8 | 1.05 | 20.61 |
| 1394 | 2 | 35.9 | 1.08 | 22.99 |
| | 3 | 51.6 | 1.08 | 23.20 |
| 1396 | 2 | 26.3 | 1.78 | 18.90 |
| | 3 | 49.3 | 1.72 | 22.04 |
| 1398 | 2 | 34.4 | 1.57 | 22.57 |
| | 3 | 51.2 | 1.24 | 23.09 |

Table 13 (Continued)

| Moving Vehicle Study Number | Microscopic Subsystem Number | Mean Velocity (ft/sec) | Acceleration Noise (ft/sec/sec) | Fuel Consumption (miles/gal) |
|---|---|---|---|---|
| 1400 | 2 | 31.7 | 1.66 | 20.55 |
| | 3 | 49.7 | 1.70 | 24.71 |
| 1522 | 2 | 81.4 | 0.96 | 38.44 |
| | 3 | 80.1 | 0.88 | 38.63 |
| 1524 | 2 | 66.4 | 0.87 | 38.68 |
| | 3 | 67.7 | 0.87 | 32.55 |
| 1526 | 2 | 50.8 | 1.27 | 44.28 |
| | 3 | 42.6 | 1.40 | 24.67 |
| 1528 | 2 | 26.3 | 1.78 | 17.78 |
| | 3 | 27.3 | 1.81 | 17.44 |
| 1530 | 2 | 35.0 | 1.56 | 20.40 |
| | 3 | 48.3 | 1.60 | 26.63 |
| 1532 | 2 | 69.8 | 1.04 | 32.06 |
| | 3 | 71.1 | 0.83 | 29.08 |
| 1534 | 2 | 77.7 | 1.37 | 36.74 |
| | 3 | 71.8 | 1.12 | 33.95 |
| 1536 | 2 | 66.4 | 0.94 | 37.65 |
| | 3 | 64.1 | 0.90 | 26.99 |
| 1849 | 2 | 75.9 | 1.23 | 32.79 |
| | 3 | 77.6 | 1.06 | 33.69 |
| 1851 | 2 | 62.2 | 1.30 | 44.65 |
| | 3 | 56.9 | 0.86 | 35.35 |
| 1853 | 2 | 37.0 | 1.64 | 32.71 |
| | 3 | 54.3 | 1.30 | 28.86 |
| 1855 | 2 | 63.9 | 1.08 | 43.56 |
| | 3 | 68.1 | 1.63 | 28.99 |
| 1857 | 2 | 82.2 | 4.29 | 29.07 |
| | 3 | 73.7 | 1.90 | 38.80 |
| 1859 | 2 | 81.8 | 2.12 | 23.14 |
| | 3 | 78.1 | 1.91 | 42.51 |
| 487 | 2 | 65.2 | 1.40 | 22.17 |
| | 3 | 63.7 | 2.91 | 21.80 |
| 1269 | 2 | 80.8 | 1.78 | 28.55 |
| | 3 | 81.8 | 1.33 | 27.33 |
| 1271 | 2 | 78.4 | 1.79 | 44.73 |
| | 3 | 70.7 | 0.95 | 34.71 |
| 1273 | 2 | 76.0 | 1.72 | 38.35 |
| | 3 | 68.1 | 1.15 | 34.73 |
| 1277 | 2 | 85.8 | 1.10 | 37.84 |
| | 3 | 85.5 | 2.04 | 35.53 |
| 1331 | 2 | 67.9 | 1.13 | 28.81 |
| | 3 | 67.0 | 1.18 | 31.02 |

Table 13 (Continued)

| Moving Vehicle Study Number | Microscopic Subsystem Number | Mean Velocity (ft/sec) | Acceleration Noise (ft/sec/sec) | Fuel Consumption (miles/gal) |
|---|---|---|---|---|
| 1337 | 2 | 63.6 | 1.16 | 30.87 |
| | 3 | 64.8 | 0.97 | 29.59 |
| 1339 | 2 | 65.7 | 1.49 | 27.95 |
| | 3 | 60.0 | 1.15 | 31.93 |
| 1420 | 2 | 80.5 | 1.09 | 45.49 |
| | 3 | 82.3 | 1.25 | 34.26 |
| 1422 | 2 | 68.6 | 1.24 | 46.89 |
| | 3 | 65.8 | 1.03 | 28.97 |
| 1424 | 2 | 81.4 | 1.09 | 32.95 |
| | 3 | 76.2 | 1.08 | 34.64 |
| 1426 | 2 | 81.0 | 1.33 | 32.73 |
| | 3 | 79.6 | 1.38 | 34.65 |
| 1428 | 2 | 79.8 | 0.98 | 37.56 |
| | 3 | 80.3 | 1.04 | 34.94 |
| 1464 | 2 | 75.4 | 0.91 | 38.00 |
| | 3 | 79.2 | 1.04 | 26.45 |
| 1466 | 2 | 83.6 | 0.99 | 29.61 |
| | 3 | 84.2 | 1.24 | 29.16 |
| 1468 | 2 | 65.7 | 1.29 | 37.31 |
| | 3 | 68.8 | 1.43 | 29.18 |
| 1821 | 2 | 62.2 | 1.12 | 31.98 |
| | 3 | 60.5 | 1.07 | 31.14 |
| 1823 | 2 | 68.0 | 1.02 | 33.00 |
| | 3 | 70.9 | 1.20 | 29.00 |
| 1825 | 2 | 73.8 | 1.14 | 37.12 |
| | 3 | 68.4 | 1.59 | 26.83 |
| 1827 | 2 | 84.0 | 1.18 | 31.76 |
| | 3 | 80.4 | 1.11 | 35.01 |

# APPENDIX D

## Empirical Results

Estimated time cost schedules for 10-13-71 A.M.

| System Speed (miles/ hour) | $\varepsilon_{tq.}$ | ASTC (min./ mile) | Time Cost Component of Optimal Toll (min./ mile) | Time Cost Component of Optimal Toll (cents/ mile) | MSTC (min./ mile) |
|---|---|---|---|---|---|
| 66 | 0.0097 | 0.9093 | 0.0088 | 0.0228 | 0.9182 |
| 65 | 0.0314 | 0.9268 | 0.0291 | 0.0755 | 0.9560 |
| 64 | 0.0436 | 0.9358 | 0.0408 | 0.1058 | 0.9767 |
| 63 | 0.0710 | 0.9543 | 0.0677 | 0.1754 | 1.0221 |
| 62 | 0.0863 | 0.9638 | 0.0831 | 0.2154 | 1.0470 |
| 61 | 0.1206 | 0.9835 | 0.1186 | 0.3073 | 1.1022 |
| 60 | 0.1607 | 1.0039 | 0.1613 | 0.4179 | 1.1653 |
| 59 | 0.1832 | 1.0145 | 0.1858 | 0.4814 | 1.2004 |
| 58 | 0.2338 | 1.0363 | 0.2423 | 0.6277 | 1.2786 |
| 57 | 0.2624 | 1.0475 | 0.2748 | 0.7119 | 1.3224 |
| 56 | 0.3268 | 1.0708 | 0.3500 | 0.9065 | 1.4208 |
| 55 | 0.4029 | 1.0951 | 0.4413 | 1.1430 | 1.5364 |
| 54 | 0.4461 | 1.1076 | 0.4942 | 1.2800 | 1.6018 |
| 53 | 0.5446 | 1.1336 | 0.6175 | 1.5993 | 1.7511 |
| 52 | 0.6009 | 1.1471 | 0.6894 | 1.7855 | 1.8365 |
| 51 | 0.7304 | 1.1750 | 0.8583 | 2.2230 | 2.0334 |
| 50 | 0.8873 | 1.2044 | 1.0687 | 2.7679 | 2.2731 |
| 49 | 0.9784 | 1.2196 | 1.1932 | 3.0906 | 2.4129 |
| 48 | 1.1919 | 1.2512 | 1.4914 | 3.8628 | 2.7427 |
| 47 | 1.4586 | 1.2845 | 1.8736 | 4.8528 | 3.1582 |
| 46 | 1.6174 | 1.3018 | 2.1056 | 5.4536 | 3.4075 |
| 45 | 2.0017 | 1.3379 | 2.6782 | 6.9366 | 4.0162 |
| 44 | 2.2359 | 1.3567 | 3.0337 | 7.8573 | 4.3905 |
| 43 | 2.8201 | 1.3960 | 3.9369 | 10.1967 | 5.3330 |
| 42 | 3.6254 | 1.4376 | 5.2119 | 13.4990 | 6.6495 |
| 41 | 4.1501 | 1.4593 | 6.0564 | 15.6861 | 7.5157 |
| 40 | 5.5839 | 1.5048 | 8.4030 | 21.7639 | 9.9079 |
| 39 | 6.5927 | 1.5286 | 10.0781 | 26.1024 | 11.6068 |
| 38 | 9.6997 | 1.5786 | 15.3127 | 39.6600 | 16.8914 |
| 37 | 16.1770 | 1.6320 | 26.4020 | 68.3813 | 28.0341 |
| 36 | 22.9365 | 1.6601 | 38.0779 | 98.6218 | 39.7380 |
| 35 | 87.2541 | 1.7192 | 150.0143 | 388.5372 | 151.7336 |
| 34 | -377.1230 | 1.7504 | -660.1350 | -1709.7497 | |

Estimated time cost schedules for 10-13-71 P.M.

| System Speed (miles/ hour) | $\varepsilon_{tq}$ | ASTC (min./ mile) | Time Cost Component of Optimal Toll (min./ mile) | Time Cost Component of Optimal Toll (cents/ mile) | MSTC (min./ mile) |
|---|---|---|---|---|---|
| 71 | 0.0144 | 0.8405 | 0.0121 | 0.0313 | 0.8527 |
| 70 | 0.0302 | 0.8524 | 0.0258 | 0.0668 | 0.8782 |
| 69 | 0.0477 | 0.8646 | 0.0412 | 0.1068 | 0.9059 |
| 68 | 0.0668 | 0.8772 | 0.0586 | 0.1519 | 0.9359 |
| 67 | 0.0879 | 0.8902 | 0.0782 | 0.2027 | 0.9685 |
| 66 | 0.1110 | 0.9035 | 0.1003 | 0.2599 | 1.0039 |
| 65 | 0.1365 | 0.9173 | 0.1252 | 0.3244 | 1.0425 |
| 64 | 0.1645 | 0.9314 | 0.1532 | 0.3970 | 1.0847 |
| 63 | 0.1954 | 0.9460 | 0.1848 | 0.4788 | 1.1309 |
| 62 | 0.2294 | 0.9611 | 0.2204 | 0.5710 | 1.1816 |
| 61 | 0.2668 | 0.9767 | 0.2606 | 0.6750 | 1.2373 |
| 60 | 0.3082 | 0.9927 | 0.3059 | 0.7925 | 1.2987 |
| 59 | 0.3539 | 1.0094 | 0.3572 | 0.9252 | 1.3666 |
| 58 | 0.4044 | 1.0265 | 0.4151 | 1.0753 | 1.4417 |
| 57 | 0.4604 | 1.0443 | 0.4808 | 1.2454 | 1.5253 |
| 56 | 0.5226 | 1.0627 | 0.5554 | 1.4385 | 1.6181 |
| 55 | 0.5917 | 1.0818 | 0.6401 | 1.6579 | 1.7219 |
| 54 | 0.6687 | 1.1015 | 0.7366 | 1.9079 | 1.8382 |
| 53 | 0.7546 | 1.1220 | 0.8468 | 2.1932 | 1.9689 |
| 52 | 0.8509 | 1.1433 | 0.9728 | 2.5198 | 2.1162 |
| 51 | 0.9589 | 1.1654 | 1.1176 | 2.8946 | 2.2830 |
| 50 | 1.0805 | 1.1884 | 1.2841 | 3.3260 | 2.4725 |
| 49 | 1.2179 | 1.2123 | 1.4765 | 3.8243 | 2.6888 |
| 48 | 1.3738 | 1.2371 | 1.6996 | 4.4020 | 2.9367 |
| 47 | 1.7542 | 1.2900 | 2.2631 | 5.8614 | 3.5532 |
| 46 | 1.9876 | 1.3182 | 2.6203 | 6.7866 | 3.9386 |
| 45 | 2.2577 | 1.3477 | 3.0428 | 7.8809 | 4.3905 |
| 44 | 2.5722 | 1.3785 | 3.5459 | 9.1840 | 4.9245 |
| 43 | 2.9412 | 1.4108 | 4.1495 | 10.7473 | 5.5603 |
| 42 | 3.3781 | 1.4445 | 4.8800 | 12.6394 | 6.3246 |
| 41 | 3.9006 | 1.4800 | 5.7730 | 14.9521 | 7.2530 |
| 40 | 4.5328 | 1.5172 | 6.8776 | 17.8129 | 8.3948 |
| 39 | 5.3089 | 1.5564 | 8.2628 | 21.4008 | 9.8193 |
| 38 | 6.2778 | 1.5976 | 10.0297 | 25.9770 | 11.6273 |
| 37 | 7.5130 | 1.6411 | 12.3298 | 31.9343 | 13.9710 |
| 36 | 9.1301 | 1.6270 | 15.4026 | 39.8929 | 17.0897 |
| 35 | 11.3200 | 1.7355 | 19.6466 | 50.8848 | 21.3822 |
| 34 | 14.4251 | 1.7869 | 25.7776 | 66.7641 | 27.5646 |
| 33 | 19.1230 | 1.8415 | 35.2161 | 91.2097 | 37.0576 |
| 32 | 26.9690 | 1.8995 | 51.2290 | 132.6831 | 53.1285 |
| 31 | 42.4958 | 1.9613 | 83.3478 | 215.8709 | 85.3091 |
| 30 | 86.8680 | 2.0272 | 176.1031 | 456.1071 | 178.1304 |
| 29 | 1116.8447 | 2.0977 | 2342.8722 | 6068.0390 | 2344.9699 |
| 28 | -118.6225 | 2.1733 | -257.8082 | -667.7234 | - |

Estimated operating cost schedules for 10-13-71 A.M.

| System Speed (miles/ hour) | $\varepsilon_{O.C.,q}$ | ASFC (gal./ mile) | Fuel Cost Component of Optimal Toll (gal./ mile) | Fuel Cost Component of Optimal Toll (cents/ mile) | Oper. Cost Component of Opt. Toll (cents/ mile) | MSFC (gal./ mile) |
|---|---|---|---|---|---|---|
| 66 | -0.0299 | 0.0443 | -0.0013 | -0.0424 | -0.0848 | 0.0430 |
| 65 | -0.0779 | 0.0422 | -0.0032 | -0.1053 | -0.2107 | 0.0389 |
| 64 | -0.1230 | 0.0403 | -0.0049 | -0.1053 | -0.3175 | 0.0353 |
| 63 | -0.1641 | 0.0385 | -0.0063 | -0.2026 | -0.4053 | 0.0322 |
| 62 | -0.2002 | 0.0370 | -0.0074 | -0.2371 | -0.4742 | 0.0295 |
| 61 | -0.2301 | 0.0355 | -0.0081 | -0.2620 | -0.5241 | 0.0274 |
| 60 | -0.2526 | 0.0343 | -0.0086 | -0.2776 | -0.5553 | 0.0256 |
| 59 | -0.2665 | 0.0332 | -0.0088 | -0.2836 | -0.5672 | 0.0243 |
| 58 | -0.2708 | 0.0323 | -0.0087 | -0.2800 | -0.5600 | 0.0235 |
| 57 | -0.2644 | 0.0315 | -0.0083 | -0.2667 | -0.5334 | 0.0231 |
| 56 | -0.2463 | 0.0308 | -0.0076 | -0.2433 | -0.4867 | 0.0232 |
| 55 | -0.2160 | 0.0303 | -0.0065 | -0.2097 | -0.4195 | 0.0237 |
| 54 | -0.1726 | 0.0299 | -0.0051 | -0.1656 | -0.3312 | 0.0248 |
| 53 | -0.1159 | 0.0297 | -0.0034 | -0.1102 | -0.2205 | 0.0262 |
| 52 | -0.0455 | 0.0295 | -0.0013 | -0.0431 | -0.0862 | 0.0282 |
| 51 | 0.0388 | 0.0295 | 0.0011 | 0.0367 | 0.0734 | 0.0307 |
| 50 | 0.1372 | 0.0297 | 0.0040 | 0.1304 | 0.2608 | 0.0337 |
| 49 | 0.2500 | 0.0299 | 0.0074 | 0.2392 | 0.4785 | 0.0373 |
| 48 | 0.3777 | 0.0302 | 0.0114 | 0.3655 | 0.7310 | 0.0416 |
| 47 | 0.5211 | 0.0306 | 0.0159 | 0.5112 | 1.0225 | 0.0466 |
| 46 | 0.6818 | 0.0311 | 0.0212 | 0.6805 | 1.3610 | 0.0524 |
| 45 | 0.8621 | 0.0318 | 0.0274 | 0.8773 | 1.7546 | 0.0592 |
| 44 | 1.0659 | 0.0325 | 0.0346 | 1.1088 | 2.2177 | 0.0671 |
| 43 | 1.2988 | 0.0332 | 0.0432 | 1.3836 | 2.7672 | 0.0765 |
| 42 | 1.5699 | 0.0341 | 0.0536 | 1.7161 | 3.4322 | 0.0877 |
| 41 | 1.8933 | 0.0351 | 0.0664 | 2.1266 | 4.2532 | 0.1015 |
| 40 | 2.2922 | 0.0361 | 0.0827 | 2.6486 | 5.2973 | 0.1188 |
| 39 | 2.8064 | 0.0371 | 0.1043 | 3.3389 | 6.6778 | 0.1415 |
| 38 | 3.5103 | 0.0383 | 0.1345 | 4.3044 | 8.6089 | 0.1728 |
| 37 | 4.5591 | 0.0395 | 0.1801 | 5.7642 | 11.5285 | 0.2196 |
| 36 | 6.3385 | 0.0407 | 0.2582 | 8.2654 | 16.5309 | 0.2990 |
| 35 | 10.1424 | 0.0420 | 0.4263 | 13.6443 | 27.2887 | 0.4684 |
| 34 | 24.6555 | 0.0433 | 1.0693 | 34.2180 | 68.4360 | 1.1126 |
| 33 | -57.2629 | 0.0447 | -2.5619 | -81.9821 | -163.9643 | - |

Estimated operating cost schedules for 10-13-71 P.M.

| System Speed (miles/ hour) | $\varepsilon_{O.C.,q}$ | ASFC (gal./ mile) | Fuel Cost Component of Optimal Toll (gal./ mile) | Fuel Cost Component of Optimal Toll (cents/ mile) | Oper. Cost Component of Opt. Toll (cents/ mile) | MSFC (gal./ mile) |
|---|---|---|---|---|---|---|
| 67 | -0.2975 | 0.0466 | -0.0138 | -0.4438 | -0.8877 | 0.0327 |
| 66 | -0.3482 | 0.0443 | -0.0154 | -0.4940 | -0.9881 | 0.0289 |
| 65 | -0.3952 | 0.0422 | -0.0166 | -0.5342 | -1.0684 | 0.0255 |
| 64 | -0.4374 | 0.0403 | -0.0176 | -0.5643 | -1.1287 | 0.0226 |
| 63 | -0.4734 | 0.0385 | -0.0182 | -0.5845 | -1.1690 | 0.0203 |
| 62 | -0.5020 | 0.0370 | -0.0185 | -0.5944 | -1.1888 | 0.0184 |
| 61 | -0.5217 | 0.0355 | -0.0185 | -0.5941 | -1.1882 | 0.0170 |
| 60 | -0.5309 | 0.0343 | -0.0182 | -0.5836 | -1.1672 | 0.0161 |
| 59 | -0.5283 | 0.0332 | -0.0175 | -0.5621 | -1.1243 | 0.0156 |
| 58 | -0.5125 | 0.0323 | -0.0165 | -0.5299 | -1.0598 | 0.0157 |
| 57 | -0.4821 | 0.0315 | -0.0151 | -0.4863 | -0.9726 | 0.0163 |
| 56 | -0.4360 | 0.0308 | -0.0134 | -0.4307 | -0.8614 | 0.0174 |
| 55 | -0.3731 | 0.0303 | -0.0113 | -0.3624 | -0.7248 | 0.0190 |
| 54 | -0.2925 | 0.0299 | -0.0087 | -0.2805 | -0.5611 | 0.0212 |
| 53 | -0.1934 | 0.0297 | -0.0057 | -0.1840 | -0.3680 | 0.0239 |
| 52 | -0.0750 | 0.0295 | -0.0022 | -0.0710 | -0.1421 | 0.0273 |
| 51 | 0.0635 | 0.0295 | 0.0018 | 0.0601 | 0.1203 | 0.0314 |
| 50 | 0.2236 | 0.0297 | 0.0066 | 0.2125 | 0.4250 | 0.0363 |
| 49 | 0.4068 | 0.0299 | 0.0121 | 0.3894 | 0.7788 | 0.0420 |
| 48 | 0.6159 | 0.0302 | 0.0186 | 0.5960 | 1.1920 | 0.0488 |
| 47 | 0.8547 | 0.0306 | 0.0262 | 0.8386 | 1.6772 | 0.0568 |
| 46 | 1.1297 | 0.0311 | 0.0352 | 1.1275 | 2.2551 | 0.0664 |
| 45 | 1.4505 | 0.0318 | 0.0461 | 1.4760 | 2.9520 | 0.0779 |
| 44 | 1.8326 | 0.0325 | 0.0595 | 1.9065 | 3.8130 | 0.0920 |
| 43 | 2.3015 | 0.0332 | 0.0766 | 2.4517 | 4.9034 | 0.1099 |
| 42 | 2.9009 | 0.0341 | 0.0990 | 3.1711 | 6.3422 | 0.1332 |
| 41 | 3.7120 | 0.0351 | 0.1302 | 4.1693 | 8.3386 | 0.1653 |
| 40 | 4.9015 | 0.0361 | 0.1769 | 5.6638 | 11.3277 | 0.2131 |
| 39 | 6.8751 | 0.0371 | 0.2556 | 8.1797 | 16.3594 | 0.2928 |
| 38 | 10.9344 | 0.0383 | 0.4190 | 13.4082 | 26.8165 | 0.4573 |
| 37 | 24.7804 | 0.0395 | 0.9790 | 31.3304 | 62.6608 | 1.0185 |
| 36 | -109.1003 | 0.0407 | -4.4458 | - | - | - |
| 35 | -17.1864 | 0.0420 | -2.7225 | - | - | - |
| 34 | -9.2918 | 0.0433 | -0.4029 | - | - | - |

Estimated toll schedules for 10-13-71 A.M.

| System Speed (miles/hour) | Time Cost Component (cents/mile) | Operating Cost Component (cents/mile) | Optimal Toll (cents/mile) |
|---|---|---|---|
| 66 | 0.0228 | -0.0849 | -0.0621 |
| 65 | 0.0755 | -0.2108 | -0.1353 |
| 64 | 0.1058 | -0.3176 | -0.2118 |
| 63 | 0.1755 | -0.4054 | -0.2299 |
| 62 | 0.2155 | -0.4742 | -0.2587 |
| 61 | 0.3074 | -0.5242 | -0.2168 |
| 60 | 0.4179 | -0.5553 | -0.1374 |
| 59 | 0.4815 | -0.5672 | -0.0857 |
| 58 | 0.6278 | -0.5601 | 0.0677 |
| 57 | 0.7120 | -0.5334 | 0.1786 |
| 56 | 0.9066 | -0.4868 | 0.4198 |
| 55 | 1.1430 | -0.4196 | 0.7234 |
| 54 | 1.2800 | -0.3312 | 0.9488 |
| 53 | 1.5993 | -0.2205 | 1.3788 |
| 52 | 1.7856 | -0.0862 | 1.6994 |
| 51 | 2.2231 | 0.0735 | 2.2966 |
| 50 | 2.7680 | 0.2609 | 3.0289 |
| 49 | 3.0906 | 0.4786 | 3.5692 |
| 48 | 3.8629 | 0.7310 | 4.5939 |
| 47 | 4.8528 | 1.0226 | 5.8754 |
| 46 | 5.4537 | 1.3611 | 6.8048 |
| 45 | 6.9366 | 1.7547 | 8.6913 |
| 44 | 7.8573 | 2.2178 | 10.0751 |
| 43 | 10.1968 | 2.7673 | 12.9641 |
| 42 | 13.4990 | 3.4323 | 16.9313 |
| 41 | 15.6862 | 4.2532 | 19.9394 |
| 40 | 21.7639 | 5.2973 | 27.0612 |
| 39 | 26.1024 | 6.6779 | 32.7803 |
| 38 | 39.6600 | 8.6090 | 48.2690 |
| 37 | 68.3814 | 11.5286 | 79.9100 |
| 36 | 98.6218 | 16.5309 | 115.1527 |
| 35 | 388.5372 | 27.2888 | 415.8260 |
| 34 | ∞ | 68.4360 | ∞ |
| 33 | | ∞ | |

Estimated toll schedules for 10-13-71 P.M.

| System Speed (miles/hour) | Time Cost Component (cents/mile) | Operating Cost Component (cents/mile) | Optimal Toll (cents/mile) |
|---|---|---|---|
| 67 | 0.2027 | -0.8877 | -0.6850 |
| 66 | 0.2600 | -0.9882 | -0.7282 |
| 65 | 0.3244 | -1.0685 | -0.7441 |
| 64 | 0.3970 | -1.1288 | -0.7318 |
| 63 | 0.4788 | -1.1690 | -0.6902 |
| 62 | 0.5711 | -1.1888 | -0.6177 |
| 61 | 0.6751 | -1.1883 | -0.5132 |
| 60 | 0.7925 | -1.1672 | -0.3747 |
| 59 | 0.9252 | -1.1244 | -0.1992 |
| 58 | 1.0754 | -1.0598 | 0.0156 |
| 57 | 1.2455 | -0.9726 | 0.2729 |
| 56 | 1.4385 | -0.8614 | 0.5771 |
| 55 | 1.6580 | -0.7248 | 0.9332 |
| 54 | 1.9079 | -0.5612 | 1.3467 |
| 53 | 2.1933 | -0.3680 | 1.8253 |
| 52 | 2.5198 | -0.1421 | 2.3777 |
| 51 | 2.8946 | 0.1234 | 3.0180 |
| 50 | 3.3260 | 0.4251 | 3.7511 |
| 49 | 3.8243 | 0.7789 | 4.6032 |
| 48 | 4.4020 | 1.1921 | 5.5941 |
| 47 | 5.8615 | 1.6772 | 7.5387 |
| 46 | 6.7867 | 2.2551 | 9.0418 |
| 45 | 7.8809 | 2.9521 | 10.8330 |
| 44 | 9.1840 | 3.8131 | 12.9971 |
| 43 | 10.7474 | 4.9035 | 15.6509 |
| 42 | 12.6394 | 6.3422 | 18.9816 |
| 41 | 14.9522 | 8.3386 | 23.2908 |
| 40 | 17.8130 | 11.3277 | 29.1407 |
| 39 | 21.4009 | 16.3595 | 37.7604 |
| 38 | 25.9770 | 26.8165 | 52.7935 |
| 37 | 31.9344 | 62.6609 | 94.5953 |
| 36 | 39.8930 | $\infty$ | $\infty$ |
| 35 | 50.8848 | | |
| 34 | 66.7641 | | |